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ELEMENTS OF PERSPECTIVE;

CONTAINING

THE NATURE OF LIGHT AND COLOURS,
&c. &c.

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ELEMENTS OF PERSPECTIVE;

CONTAINING

THE NATURE OF LIGHT AND COLOURS,

AND THE

THEORY AND PRACTICE OF PERSPECTIVE,

IN REGARD TO LINES, SURFACES, AND SOLIDS, WITH ITS APPLICATION
TO ARCHITECTURE.

TO WHICH ARE ADDED

RULES

FOR

PAINTING IN TRANSPARENT WATER COLOURS.

BY JOHN WOOD,

MASTER OF THE DRAWING ACADEMY ESTABLISHED AT EDINBURGH BY THE
HONOURABLE THE BOARD OF TRUSTEES FOR MANUFACTORIES, &c.

LONDON:

PRINTED FOR
G. CAWTHORN, BRITISH LIBRARY, NO. 132, STRAND,
BOOKSELLER TO HER ROYAL HIGHNESS THE PRINCESS OF WALES:
MESSRS. RICHARDSON, ROYAL-EXCHANGE; H. D. SYMONDS, J. WALLIS, WEST AND
HUGHES, PATERNOSTER-ROW; J. WRIGHT, PICCADILLY; AND
P. HILL, EDINBURGH.

1799.

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PREFACE.

The Art of Perſpective has been regarded by ſome as a branch of optics, by others as a problem in geometry; its firſt principle, which is purely optical, being once admitted, all its operations are afterwards geometrical. This art undoubtedly owes its origin to painting, and particularly to the decorations of the theatre, in which there was a neceſſity of finding a method to repreſent objects upon the ſame plane, ſo as to have the appearance of being of different magnitudes, and at different diſtances from the eye of the ſpectator. It would naturally occur to an obſerving painter, that two parallel lines, ſuch as an avenue of trees, appear to approach each other, in proportion to their length; that a level plain appears gradually to aſcend as it extends from the eye; and that a plane, though parallel to the horizon, if above the eye, ſuch as the ceiling of a long room, would have the appearance of deſcending, or being lower at that end moſt remote from the eye. Such, no doubt, would be the leading obſervations that would guide painters in the execution of their ſcenes; but mathematicians, who are never ſatisfied but with the moſt rigorous exactneſs, inveſtigated the cauſes of theſe effects, and the means of producing them, which forms the Syſtem of Perſpective.

The general principle which both the ancients and moderns have proceeded upon in forming theſe rules, is the ſame they ſuppoſed the plane ſurface of a tablet, ſuch as a plate of glaſs, to be placed between their eye and the objects which they wiſhed to repreſent, and that traces of theſe objects were formed upon this plane by the rays of light emitted from them to the eye; which, if coloured in the ſame manner as the objects themſelves, would produce the ſame effect.

The Art of Perſpective, therefore, conſiſts in determining geometrically thoſe points where the rays of light proceeding to the eye cut the plane, which in reality is only to find the projection of theſe objects from which the rays proceed. From Vitruvius (*Archit.* B. VII. c. 1.) we learn that Æſchylus inſtructed one Agatharcus how to prepare a tragic ſcene for one of his plays, which was to be exhibited at Athens, and that he publiſhed a treatiſe upon the ſubject, and inſtructed two of his pupils, Democritus and Anaxagoras, who afterwards proſecuted the ſubject farther. Montucla ſays (Part III. Lib. V.), that among the writings of Anaxagoras there was a book entitled, *Actinographia*, or *Radiorum Deſcriptio*, which is probably the ſame that Vitruvius refers to. The treatiſes of Euclid and Heliodorus Lariſſeus, which are called Perſpective, contain only ſome general elements of optics, and can by no means be applied to practice.

Geminus of Rhodes, a celebrated mathematician in Cicero's time, is ſaid alſo to have written upon this ſcience.

It is certain, however, that the Romans were acquainted with the rules of Perſpective, from Pliny's account (*Nat. Hiſt.* Lib. XXXV. cap. 4.) of the repreſentation of the ſcenes in the plays given by Claudius Pulcher, by which birds were ſo deceived as to ſettle on the fictitious roofs. But, of the theory of this art among the ancients, we are quite ignorant, as none of their writings have eſcaped the general devaſtation of literature in the dark ages of Europe. We may therefore juſtly conſider the moderns as the ſecond inventors and revivers of this art, which commenced with painting and ſculpture, in the middle of the 15th and beginning of the 16th centuries.

The firſt authors who laid down rules of Perſpective were Bartolomea Bramantino of Milan, whoſe book, *Regole di Perſpectiva, e Miſure delle Antichitta de Lombardia*, was publiſhed 1440, and Pietro del Borgo San Stephano, another Italian, whoſe treatiſe, in three books, Ignatius Danti highly praiſes. Albert Durer, a German, conſtructed a machine about the ſame time, upon the principles of Borgo, by which he could trace the perſpective appearance of objects. After this author, Daniel Barbaro of Aquila publiſhed a treatiſe in 1569. Balthazar Peruzzi of Siena, who died in 1536, was, however, the moſt happy among the Italians in giving an elegant and perſpicuous treatiſe on this ſubject, after the principles of Borgo, and had the merit of inventing what are called Points of Diſtance. Vignola, in his Treatiſe on Perſpective, has exactly copied Borgo; and has the advantage of having a learned commentary, which is added to it by Ignatius Danti.

Guido Ubaldi, another Italian, treated of Perſpective in a ſtill more intelligent and ſcientific manner than any of the authors already mentioned, and may be regarded as the firſt who diſcovered all the general principles of this ſcience. He found out this moſt uſeful propoſition, that all lines parallel to one another, if they be inclined to the ground line, converge to ſome point in the horizontal line; and that through this point alſo will paſs a line drawn from the eye parallel to them. His Perſpective was printed at Piſaro, in 1600, and contained the firſt principles of the method afterwards publiſhed by Dr. Brook Taylor. After this period, in the laſt century, ſo many treatiſes appeared on this ſubject, that it would be almoſt endleſs to give a liſt of their various authors, very few of which have met with the public regard. Some were old authors reviſed, or the productions of perſons who did not ſeem to comprehend what were the proper materials of which ſuch works ought to be compoſed. Thoſe of them which were held in higheſt eſtimation were the Inſtitutes of Perſpective, by Hondius; Leſſons of Perſpective, by Ducerceau; and a Treatiſe by Father Dubreuil. Since that time, this Art has been gradually improved by ſubſequent geometricians, particularly by Dr. Brook Taylor, whoſe principles are far more general than thoſe of any of his predeceſſors. He did not confine his rules, as they had done, to the horizontal plane only, but made them general, ſo as to affect every ſpecies of lines and planes, whether they were parallel to the horizon or not. Beſides, his ſyſtem is the only one calculated for artiſts and practitioners in the art of deſign. It explains alſo the perſpective of ſhadows, the reflections of objects from poliſhed planes, and the inverſe practice of Perſpective. His Linear Perſpective was firſt publiſhed in 1715, and his New Principles of Linear Perſpective in 1719, which he intended as an explanation of his firſt Treatiſe. There have been ſince good Treatiſes alſo wrote by Meſſrs. Hamilton, Cowley, Emerſon, and Kirby.

THE following Elements were drawn up chiefly with the intention of ſerving as a text book to my own Scholars, and for the uſe of thoſe perſons who might have a deſire of acquiring a knowledge of the principles upon which Perſpective is founded, without entering into a variety of examples or a detail of particular proceſſes, which is the caſe with moſt of the beſt authors on this ſubject.

In compoſing it, I have freely made uſe of all ſuch former treatiſes on the ſubject as were fit for my purpoſe, and in particular Mr. Hamilton's Stereography, which is the beſt complete ſyſtem for all the branches of perſpective I have ſeen. The part regarding Architecture, as it conſiſts only of mechanical rules, founded upon the theory, is the ſame as in Mr. Kirby's Elements.

To eſtabliſh the Art of Perſpective entirely upon geometrical principles, ſo as to render the theory more eaſy, and the practice more general and extenſive,, is what I have endeavoured to do; in which, if I have ſucceeded, I do not repent the labour I have beſtowed.

There have, no doubt, been many elementary books publiſhed with the ſame intention; but, as they are deſigned for perſons unacquainted with the mathematics, their authors have of courſe been under the neceſſity, inſtead of explaining the real principles of Perſpective, only to furniſh the ſtudent with the practical method of performing ſome of the eaſy problems in this art; which knowledge they regard as the only part neceſſary to be known for the art of deſign. And indeed the artiſts themſelves have in general an unfortunate idea, that a knowledge of the mathematics has only a tendency to cramp the freedom of the imagination, and that geometry furniſhes rules that are only ſo many fetters to men of real genius. They aſſert alſo, that thoſe perſons who diſtinguiſh themſelves by a ſtrong predilection for mathematical purſuits have their views and ſentiments on other ſubjects remarkably narrow and contracted, their general habits of thinking illiberal and vulgar, and that the mathematical ſciences, however important they may be deemed, in fact terminate in ſpeculation. This will be found to be the general language of thoſe men who have paid but a ſlight attention to ſubjects of this kind, and are therefore very willing to perſuade themſelves that the cloud in which they are involved neceſſarily ariſes from the ſubject itſelf.

It would be by no means adviſable that an artiſt ſhould enter into thoſe parts of the mathematics which are of an abſtruſe and a recondite nature; for this again, by diſtracting his attention, would diſqualify him for the practical part of his profeſſion, and make him loſe ſight of his principal object: But, making the elements of geometry the favourite recreation of his leiſure hours, would tend both to improve and enlarge his mind, and render the productions of his imagination more regular and juſt.

It is curious and entertaining to obſerve the many various ſchemes, methods, and tricks thoſe artiſts fall upon, who have no other guide but the wandering ideas of an exuberant fancy, and who ſuppoſe their art to be of too delicate and aerial a nature to admit of any fixed principles, or to be regulated by any ſtandard. Always apt to overrate their own abilities, and miſtaking the moſt trifling inventions for diſcoveries of moment, they are continually obtruding upon the public different ſchemes and ſyſtems of deſigning and colouring, that ſerve no other purpoſe, but to ſpend their own time, and amuſe their acquaintances, often as ignorant as themſelves.

So far, therefore, from agreeing with thoſe perſons who have endeavoured to make the practice of Perſpective familiar to every capacity, without mathematical knowledge, they have, in my opinion, not only rendered this uſeful art intricate, puzzling, and diſcouraging to the learner, but have done a manifeſt injury to the art of painting itſelf. For there is no art that I know of, more dependent upon geometry than that of painting. Since lines being what terminate the objects, and diſentangle the colours, it furniſhes us with the rules by which we work, and ignorant of which the ſtrongeſt intellect may be fruitleſsly or deviouſly employed.

There is another idea which is very prevalent, and, I am ſorry to ſay, the opinion even of artiſts in general, which is, that the art of painting depends entirely upon that power or faculty of the mind which is generally known under the name of Genius.

To ſuppreſs notions of this kind ought to be the particular buſineſs of every teacher; for they are not only detrimental to the art itſelf, but, if once received, will be the means of preventing even ſtudents of real merit from ever making any progreſs in the more valuable parts of their profeſſion.

The term Genius, like all other figurative terms, is not extremely accurate, and is far from being a determinate idea in the minds of moſt men. It has therefore been deſcribed under various forms and ſhapes, and for the moſt part enveloped in language both myſterious and incomprehenſible.

Men have, however, in different ages and in different places, always diſagreed in their opinions about that particular effort of the mind, and degree of excellence, that ſhould be the characteriſtic mark of this faculty. In the early and uncultivated ages, when ſociety was in its infant ſtate, genius was imagined to mark out the barbarian who could wield the ſpear with the moſt eaſe, throw the dart with the greateſt dexterity, or outſtrip his companions in the chaſe. In like manner, the Indian ſavage who ſhows moſt taſte in painting his body, or decorating his limbs with the moſt curious ſhells and feathers, is by his countrymen at this day called a man of genius; or if he ſhould employ himſelf in the conſtruction of a new canoe, or a more commodious wigwam, he receives the ſame appellation. But by degrees, as mankind began to emerge from a ſtate of the utmoſt ignorance and barbarity, and the arts began to unfold themſelves, the diſcovery of the moſt ſimple problem in geometry, or the power of drawing the likeneſs of any object, would be conſidered as the greateſt effort of genius. In proceſs of time, however, the name of genius ſhifted its application, and was given only to thoſe philoſophers who attempted to inveſtigate the cauſes of the various phenomena in nature; to thoſe orators who diſplayed, in a graceful elocution, an impetuous ardour in their delivery; and to thoſe artiſts who, to the likeneſs of the object they repreſented, added alſo expreſſion, grace and dignity.

What is now called genius, is confined to thoſe few enlightened minds deſtined by Providence to enlarge the ſphere of human knowledge, by exploring unbeaten tracks, and perfecting the productions of art; ſo that we muſt infer a period will arrive, when genius will no longer have an opportunity of diſplaying its wonderful powers over the minds of men.

There are ſome authors who, by following fancies more than facts, and taking the conceits of their own imaginations rather than the records of hiſtory for their guides, confine genius to the fine arts alone, which, they ſay, are of a divine nature, formed by the hands of the Creator, and appeared the ſame day man put his foot upon the earth; conſequently, were in the higheſt perfection in the rudeſt and earlieſt ages. Poſitions ſo wild and extravagant as theſe, it would not only be abſurd, but ſerving little purpoſe to attempt to confute, as nothing is more evident than that the arts of elegance and refinement muſt have originated long after the neceſſary arts were known; and that they can never arrive to any degree of perfection but among an enlightened and civilized people.

Therefore, without entering farther into the various hypotheſes and opinions which have been framed to account for the powers of genius, or that unequal diſtribution of talents among mankind which is evident to every obſerver, none of which are any way ſatisfactory, and, when examined, only prove that no individual is, by nature, more adapted for one purſuit than another, but that all the arts and ſciences equally preſent a ſphere for his exerciſe, and afford ſcope to his exertion. We may conclude, that the progreſs or degree of perfection which any ſtudent makes in this or any other profeſſion, entirely depends upon the degree of exerciſe and attention which he pays to it; and that he who cultivates this art upon proper principles, and habitually, will acquire a readineſs of practice, and ſoundneſs of judgment, never to be attained by one who has not taken thoſe methods; for if his genius decides quickly, it is always uncertainly, and his quickneſs is only owing to preſumption and raſhneſs, and not to any particular power, which, in a moment, diſpels all darkneſs from his mind.

were in the highest perfection among [illegible] and earlier Ages. [illegible] to write and [illegible] as they, it would not only be absurd, but foreign to our purpose to attempt to confute; as nothing is more evident than that the arts of elegance and refinement must have originated long after the necessary arts were known; and that they can never arrive to any degree of perfection [illegible] among an enlightened and civilized people.

THERE are many whimsical [illegible] into theories, hypotheses and designs which have been framed to account for the powers of genius, or that unequal distribution of talents among mankind which is evident to every observer, none of which are any way satisfactory; and, when examined, only prove that nature [illegible] more adapted one man than another, but that all the arts are [illegible] a sphere [illegible] and [illegible] We may conclude, that the [illegible] degree of perfection which every student arrives to in the arts of imitation, entirely depends upon the [illegible] exercise and attention which he pays to [illegible] and that he who cultivates this art upon proper principles and [illegible] will acquire a [illegible] of [illegible] never to be attained by one who has not taken those methods; [illegible] genius and [illegible] always uncertain [illegible] its excellence only owing to a [illegible] and not to any [illegible] moments [illegible] from his mind.

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ELEMENTS OF PERSPECTIVE.

BOOK I.

OF OPTICS.

CHAP. I.

Of the Nature of Light.—Deſcription of the Eye.—Of Direct Viſion.

Definition I.*

Light is that principle by which objects are made perceptible to our ſenſe of ſeeing; and is compoſed of ſmall and diſtinct particles of matter, which, being inceſſantly thrown out from luminous ſubſtances, impreſs upon our organs of ſight that peculiar motion which excites in our minds the ſenſation of light and colours.

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* The nature of light was early a ſubject of ſpeculation to philoſophers. Empedocles and Plato ſuppoſed that viſion was cauſed by ſomething emitted from the eye, which, meeting with ſomething elſe that proceeds from the object, was thereby reflected back again; while Pythagoras aſcribed this effect ſolely to particles continually flying off from the ſurfaces of bodies, and entering the pupil of the eye.

Deſcartes and his followers maintained, that light was an inviſible fluid, preſent at all times, and in all places; but which required to be ſet in motion by the preſſure of a luminous body, in order to render objects viſible. This hypotheſis was amended in ſeveral reſpects by Malbranehe, who explained the nature of light, from a ſuppoſed analogy between it and ſound. He ſuppoſes that all the parts of a luminous body are in a rapid motion, which, by very quick pulſes, conſtantly compreſs the ſubtle matter between it and the eye. As theſe vibrations are greater, the body appears more luminous; and as they are quicker or ſlower, it aſſumes this or that colour. The later Carteſians, again, imagined that the fluid by which light is tranſmitted, was elaſtic; and M. Huygens ſuppoſed it proceeded in elliptical waves, and not circular; and, upon this ſuppoſition, he endeavours to explain ſeveral of the phenomena of light.

II.

A Ray of Light is an exceeding ſmall part of it, moving in a ſtraight line, which may be ſtopt or propagated alone, without the reſt of the light.

III.

A Pencil of Rays is a portion of light of a conical form, either proceeding from a point, or tending towards a point.

Theſe hypotheſes, although they retained their credit for ſome time, are entirely laid aſide. And there have been no advocates for the Carteſian philoſophy, or any method ſimilar to it, for many years.

The Newtonian theory, which is now univerſally eſtabliſhed, ſuppoſes light to conſiſt of a great number of very ſmall particles that are thrown off, by a repulſive power, from the luminous body, with an immenſe velocity, and in all directions; but which proceed in ſtraight lines, until they are turned out of their path, either by inflection, reflection, or refraction, or by being ſtopped by ſome ſubſtance which they cannot penetrate.

The velocity of the particles of light has been meaſured in various ways. The attempt was firſt made by Galileo, who placed two men with covered lights, at a mile's diſtance from each other; one of whom was to obſerve when the other uncovered his light, and that moment to exhibit his. The members of the Academy Del Cimento repeated the ſame experiment; but the reſult of both was, as may be naturally imagined, very inaccurate.

About 1672, Roemer, a Daniſh aſtronomer, being employed in making obſervations at Paris, along with Meſſrs. Picard and Caſſini, obſerved, when Jupiter was at his greateſt diſtance from the earth, the emerſions of his firſt ſatellite happened conſtantly 15 or 16 minutes later than the calculation gave them. Hence he concluded, that the light reflected by Jupiter, took up this time in

IV.

Parallel Rays are thoſe that proceed in parallel directions. Converging rays tend towards a point, and diverging rays flow from a point.

V.

A Medium is whatever the rays of light paſs through, as water, oil, glaſs, air, or any tranſparent body, and even a vacuum.

VI.

Reflection is when a ray of light flies off from the ſurface upon which it falls, without entering into it.

VII.

Refraction is the bending of a ray of light, when it goes out of one medium into another.

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running over the exceſs of diſtance; and conſequently, that it took up 16 or 18 minutes in running over the diameter of the earth's orbit; and from thence calculated its velocity to be nearly two hundred thouſand miles in a ſecond of time. This diſcovery was at firſt much diſputed, but afterwards fully confirmed by Dr. Bradely, in a ſeries of obſervations he made, to diſcover the laws of the alterations of the fixed ſtars, from the progreſſive motion of light, combined with the earth's annual motion round the ſun. The account of which he publiſhed in 1727.

The materiality of light has alſo been proved by many experiments, which ſhow that the colour and texture of bodies are changed, by being expoſed to it. And attempts have even been made to determine the momentum or force of the particles of light, in giving motion to light bodies. The moſt curious and accurate experiment of this kind ever performed, was made by Mr. Michell, and is related by Dr. Prieſtley, in his Hiſtory of Optics. It has, however, been much doubted whether the effects produced in theſe experiments, were not occaſioned by the air being heated, from the burning glaſſes made uſe of to collect the rays of light.

VIII.

A Focus is a point where reflected or refracted rays meet together. That point from which the rays ſeem to diverge, is called the Virtual Focus. The point to which parallel rays are reflected or refracted, is the Principal Focus.

IX.

The Focal Diſtance, is the diſtance of the focus from the reflecting or refracting ſurface.

X.

Catoptrics is that part of optics which relates to ſeeing, or making the pictures of objects by reflected light.

XI.

Dioptrics treats of refracted light, and every thing that relates to ſeeing through different mediums, and the pictures of objects formed by refraction through them.

XII.

A Lens is a thin piece of glaſs, or any other tranſparent ſubſtance, and of ſuch a ſhape, that the ſides of it are either flat, or the ſegments of ſpherical ſurfaces. It is conceived to be generated by turning the figure ABCD (fig. 1. 2, 3, &c.) round about the line EF, drawn through the middle of it, perpendicular to both its ſides. Fig. 1. plate 1. repreſents a plano convex lens; fig. 2. a plano concave; fig. 3. a double convex;

fig. 4. a double concave; and fig. 5. and 6. two concave convex lenſes, whereof the firſt is commonly called a Meniſcus.

XIII.

The Axis of a lens is the line E F produced, which paſſes through G and H, the centres of its ſurfaces.

XIV.

The Vertexes of a lens are the points E and F, where the axis cuts the ſurfaces; and the middle point between them is called its Centre.

XV. Fig. 7, 8.

A glaſs Priſm is a body of the form of a wedge, which has three edges, A a, B b, C c, and is bounded by two equal and parallel triangular ends, A B C and a b c, and three well poliſhed plane ſides, as A B b a, A C c a, and B C c b. When it is viewed endways, it is repreſented only by a triangle, as A B c in fig. 8.

PROPOSITION I.

When a ray of light falls obliquely upon a ſmooth ſurface, and is reflected from it, the angle of reflection is equal to the angle of incidence. Thus, if the line A B (fig. 9.) repreſents the ſurface of ſtagnating water, P C a line perpendicular to the ſurface, R C a ray of light which falls upon the ſurface at the point C, and is reflected from it in the line C E, then R C P, the angle of incidence, is equal to E C P, the angle of reflection.

The truth of this may be proved by the following ſimple experiment. Upon a ſmooth board, DEFG (fig. 11.), from a centre C, deſcribe a circle, PROS, and having drawn two diameters, PO and RS, perpendicular to each other from the point P, cut off two equal arches, PH, PK, and draw the lines CH, CK, then fix three pins perpendicular to the board at the points, C, H, and K. This being done, immerge the board into water as far as the line RS, and, holding it in a perpendicular direction to the ſurface AB, look along the pins H, C, and the image of the pin K will be ſeen in the line HC produced; which ſhows, that the ray proceeding from the pin K is reflected from the ſurface of the water along the line CH. Therefore, &c. Q. E. D.

Cor. I. RC the incident ray, and CE the reflected ray, are equally inclined to the reflecting plane; that is, the angles ACR and BCE are equal to each other.

Cor. II. When the incident ray is perpendicular to the reflecting ſurface, it is reflected directly back again along the ſame perpendicular.

Prop. II. Fig. 10.

If the ray of light that came along RC goes into the water at C, it will not proceed ſtraight forward, but will be refracted at C along a line CE, inclined to the perpendicular PCO at a leſſer angle ECO and EG, the ſine of this angle, will always have the ſame ratio to RF, the ſine of the angle of inclination.

If the refraction be made out of air into water, the ſine of incidence is to the ſine of refraction as 4 to 3;

but if out of air into glaſs, it will be nearly as 17 to 11, or as 3 to 2.

For taking the ſame board as before, produce HC to L, draw LI perpendicular to the diameter PO, from IL, cut off IQ, equal to three-fourths of IL, and draw NQ parallel to PO, and MN parallel to IL; then fixing a pin at N, dip the board into water as before, the pin at N will appear to be in the ſame line with the pins at H and C. Therefore the ray proceeding from N is ſo refracted at C, as to go to the eye along the line CH; wherefore, when the refraction is made out of air into water IL, the ſine of the angle of inclination, is to MN, which is equal to the ſine of the angle of refraction as 4 to 3. Q. E. D.

Cor. I. Refraction out of a rarer medium into a denſer, is made towards the perpendicular, or the angle of refraction is then leſs than the angle of incidence.

Cor. II. According as the angle of incidence RCP is increaſed or diminiſhed, the angle of refraction ECO is alſo increaſed or diminiſhed; and when the incident ray coincides with the perpendicular CP, it will proceed forward without any refraction.

Cor. III. While the angle of incidence RCP increaſes, the refracted ray CE is more and more bent from the courſe of the incident ray; that is, the angle DCE continually increaſes.

PROP. III. * Fig. 13.

The light of the ſun conſiſts of a mixture of ſeveral

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* This celebrated propoſition, the foundation of the theory of light and colours, was diſcovered by Sir Iſaac Newton in *1666*. The refracting angle of

ſorts of coloured rays, which are differently refrangible.

In a very dark chamber make a round hole, F, in the window ſhutter EG, about 1-4th inch in diameter. Place a glaſs priſm, ABC, very near the hole, and holding it parallel to the horizon, with its axis perpendicular to the beam SF, turn the priſm ſlowly about its axis, until the refracted light on the oppoſite wall, or the coloured image of the ſun, appears firſt to deſcend, and then to aſcend. Between the deſcent and aſcent, when the image ſeems ſtationary, ſtop the priſm, and fix it there; for in that poſture the refractions of

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the priſm he uſed in this experiment, was 64 degrees. At the diſtance of 18½ feet from the priſm, the length of the image was 10¼ inches, and its breadth 2⅛ inches. He found alſo, that in a priſm whoſe angle was leſs, the length of the image became ſhorter, but the breadth remained the ſame. When Sir Iſaac Newton tried this experiment the firſt time, he was uncertain whether or not different colours might ariſe from the light being dilated by ſome unevenneſs in the glaſs, or ſome other accidental irregularity. To be certain of this, he took another priſm, like the former, and placed it in ſuch a manner, as that the light paſſing through them both, might be refracted contrary ways, and ſo be returned by the latter into the ſame courſe from which it had been diverted by the former. In this manner he thought that the regular effects of the firſt priſm would be deſtroyed by the ſecond; but that the irregular ones would be augmented by the multiplicity of refractions. The event, however, was, that the light which was by the firſt priſm diffuſed into an oblong form, was by the ſecond reduced into a circular one, with as much regularity as if it had not paſſed through either of them. To confirm his theory ſtill further, he took two boards, and placed one of them cloſe behind the priſm at the window, ſo that the light might paſs through a ſmall hole made in it for the purpoſe, and fall on the other board, which he placed at the diſtance of 12 feet, having firſt made a ſmall hole in it alſo, for ſome of that incident light to paſs through. He then placed another priſm behind the ſecond board; ſo that the light which was tranſmitted through both the boards, might paſs through that priſm alſo, and be again refracted, before it arrived at the wall. This being done, he took the firſt priſm in his hand, and turned it about its axis, ſo much as to make the ſeveral parts of the image caſt on the ſecond board, ſucceſſively to paſs through the light on the two ſides of the priſm are equal to one another.

The priſm remaining in this ſituation, place a ſheet of white paper, MN, on the oppoſite wall, ſo that the refracted light may fall perpendicularly upon it, and an image, PT, of the ſun will be formed on the paper, of an oblong form, terminated by two rectilineal and parallel ſides, and two ſemicircular ends. This image PT will be coloured; and the colours will lie from the top at P to the bottom at T, in the following order: violet, purple, blue, green, yellow, orange, and red, with all their intermediate degrees, in a continual ſucceſſion, perpetually varying; ſo that the moſt refracted part at P is violet, and the leaſt refracted at T is red. Therefore the light of the ſun, which appears to us white and perfectly homogeneal, is compoſed of

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the hole in it, that he might obſerve to what place on the wall the ſecond priſm would refract them; and he ſaw, by the change of thoſe places, that the light tending to that end of the image towards which the refraction of the firſt priſm was made, did, in the ſecond priſm, ſuffer a refraction conſiderably greater than the light which tended to the other end. Theſe, and ſeveral other ingenious experiments which he made with the priſm, and which are to be found in his Optics, confirmed him in the opinion that light is not homogeneal, and that the different colours of bodies ariſe from their reflecting ſome rays more copiouſly than others.

Having thus obtained a rational theory for the colour of bodies, his next inquiry was to diſcover the cauſe of their reflecting one kind of rays more than another. This he attributed to the denſity of bodies, or the magnitude of their component particles; for, in compreſſing two priſms hard together, in order to make their ſides touch each other, which chanced to be a little convex in the place of contact, they were as tranſparent as if they had been but one piece of glaſs. He remarked alſo, that round the point of contact where they were a little ſeparated from each other, rings of different colours appeared. To elucidate this matter ſtill farther, he got two convex glaſſes of a teleſcope; and where they were preſſed together, there appeared ſeveral circles or rings of corays of different colours, and endued with different degrees of refrangibility.

Cor. I. Every ray of light may be ſeparated into the ſeven primary colours; each of which has a peculiar degree of refrangibility, which cannot be changed by reflection or refraction, but remains invariably the ſame.

Cor. II. By compounding any two of the primary colours, as red and yellow, or yellow and blue, the intermediate colour, orange or green, will be produced.

Cor. III. The colours of bodies ariſe from their diſpoſitions to reflect one ſort of rays, and to abſorb others. Thoſe that reflect the leaſt refrangible rays, appearing red, and thoſe that reflect the moſt refrangible, violet.

Cor. IV. The whiteneſs of bodies ariſes from their diſpoſition to reflect all the rays of light promiſcuouſly.

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lours, different, and of various intenſities, according to their diſtance from the point of contact.

As he found the colours, therefore, to vary according to the different diſtances between the glaſs plates, he concluded that they proceeded from the different thickneſs of the plate of air intercepted between the glaſſes, which was diſpoſed according to its thinneſs or thickneſs, to reflect or tranſmit the rays of this or that particular colour. And by a great variety of ſuch other experiments and obſervations upon them, he deduced his theory of colours; by which it appears that every ſubſtance in nature is tranſparent, provided it be made ſufficiently thin. For gold, the denſeſt ſubſtance we know of, when reduced into thin leaves, tranſmits a bluiſh green light through it.

If we ſuppoſe, therefore, any denſe body to be divided into a vaſt number of extremely thin plates, the greateſt part of the rays will therefore paſs through the upper plates; and when they loſe their force, will be reflected from the under ones. Therefore, according to the number of plates through which they have paſſed, the object will aſſume this or that colour; juſt as the rings of colours appeared different in the experiment of the two plates, according to their diſtance from one another, or the thickneſs of the plate of air between them.

Cor. V. The blackneſs of bodies ariſes from their incapacity to reflect any of the rays of light, but abſorbing all the light that falls upon them.

PROP. IV. Fig. 12.

If a room be quite dark, and a ſmall hole be made in the window ſhutter, the rays proceeding from the objects without will form their pictures on a paper placed on the oppoſite wall, in their proper colours, but in an inverted poſition.

Let FGH repreſent the darkened room, E the hole in the window ſhutter, and ABCD the object without. Therefore, ſince the rays which proceed from the top of the object A, and thoſe from the bottom C, croſs each other in ſtraight lines at the hole E, the image of A will be at the bottom of the wall at a, and that of C towards the top at c. In like manner, the rays proceeding from the right hand of the object at D will form its picture on the left of the wall at d, and the image of B will appear on the right at b; and, in the ſame way, the pictures of any other object expoſed to the hole will appear upon the wall inverted. Therefore, &c. Q. E. D.

Cor. If the hole E be very ſmall, the pictures of the objects will be very diſtinct, but hardly viſible for want of light; and if the hole be large, the pictures will be brighter, but not ſo diſtinct *.

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* After this principle, is made that optical inſtrument which is called the Camera Obſcura.

Definition XV.

OF THE EYE.* Fig. 14.

ABCKL repreſents the figure of the eye, which is ſpherical, and on that account eaſily moved any way in its ſocket.

The external part ABC is a ſtrong convex pellucid ſubſtance, and is called the Cornea, becauſe, when dried, it has ſome reſemblance to a piece of tranſparent horn. The remainder of this coat, which is called the Sclerotica, viz. ALKC, is opaque, and a portion of a larger ſphere.

Immediately behind the cornea, there is a fine clear humour, which is called the Aqueous Humour, and is contained in the ſpace between ABC and EF. In this ſpace there is a membrane called the Uvea, with a hole in the middle at D, called the Pupil, of a muſcular contexture, for altering the dimenſions of that hole to admit a proper quantity of light.

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* Although the above is the general received opinion of the manner in which viſion is performed, yet, how this is effected, is a problem which has engaged the attention both of anatomiſts and opticians. The Abbe Mariotte maintained that viſion was not made in the retina, but in the choroides. As a proof of which, he deſcribes an experiment (Phil. Tranſ. No. 35.); by which he ſhows, that any object whoſe picture falls upon the baſe of the optic nerve, where it enters the bottom of the eye, is not perceived while the other eye is ſhut. And he hence infers, that if viſion was made in the retina, it would be made wherever the retina was. M. Picard alſo propoſed a way by which an object may be loſt, while both eyes are open; which he deſcribes as follows, in the ſame number of the Philoſophical Tranſactions: "Faſten againſt a wall a round "white paper of the bigneſs of an inch or two; and on the ſides of this paper "put two marks, one on the right, the other on the left ſide, each about two "feet diſtant from the paper, and ſomewhat higher. Then place yourſelf di-"rectly before the paper, at the diſtance of nine or ten feet, and put the end of "your finger over againſt both your eyes, ſo that it may hide from the right eye "the left mark, and from the left eye the right mark. If you remain firm in "that poſture, and look ſteadily with both eyes on the end of your finger, the "paper, which is not at all covered thereby, will totally diſappear." This experiment, which Mr. Picard imagined as new, was known long before, and is explained by Aguilonius, in his Fourth Book of Optics, publiſhed at Antwerp, in 1613, in treating of the horopter, wherein he ſhows, that not only one ob-

Behind this membrane is placed a ſubſtance, EF, of the form of a lens, and of a conſiderable conſiſtence, which is called the Cryſtalline Humour, from its tranſparency. This is contained in what is called the Tunica Choroides, and is ſuſpended in the middle of the eye by a ring of fibres called the Ligamentum Ciliare, as at E and F, which ſerves to adjuſt its diſtance from the bottom of the eye, ſo as to bring the image op of an object upon the retina, which is nothing

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ject, but even more may be loſt, while both eyes are open, provided they are ſeen out of the plane of the horopter.

The inference which Mariotte drew from theſe experiments, was diſputed then both by Mr. Piquet and M. de la Hire; the latter of whom has obſerved, that although the retina is the principal organ of ſight, yet, being an expanſion of the optic nerve, it is of too tender and delicate a nature, to be expoſed to the naked impreſſions of external bodies, and therefore receives the impreſſions of light from an intermediate organ, which is the choroides that receives them immediately from the object itſelf (Traité des differens accidens de la Veüe).

It has alſo been ſaid by ſome anatomiſts, in order to account for this, that the part of the optic nerve which is immediately at its entrance into the eye, forms no part of the retina, and therefore is incapable of producing viſion.

There is another queſtion concerning viſion, which has been the ſubject of much controverſy, and which ſeems ſtill to remain unſettled; and that is, What is that change which is made in the conformation of the eye, in order to obtain perfect viſion at different diſtances?

Kepler was of opinion, that the eye, in a ſtate of inaction, is accommodated to ſee with perfect diſtinctneſs at the greateſt diſtance; and that, in order to view near objects, the globe of the eye is compreſſed by its muſcles into an oblong figure, ſo as to render the axis ſo much longer as is neceſſary to unite the pencils into points upon the retina. Dr. Jurin (Eſſay upon Diſtinct and Indiſtinct Viſion, p. 137.) urges, however, as an objection againſt this hypotheſis, more than a delicate membrane, formed of the optic nerve, that enters the eye from the brain at LK, and expands itſelf over all the interior part from E to F.

The ſubſtance which fills this cavity EFKL of the eye, is called the Vitreous Humour, and bears a great reſemblance to the white of an egg.

Viſion is cauſed, therefore, in the following manner. Let OP be any diſtant object, then a pencil of rays proceeding from any point R in it will fall on the cornea ABC, and will be refracted by the aqueous humour under it to a point in the axis of that pencil produced, which, by the proportion between the ſine of incidence in air, and that of refraction in the aqueous humour, being nearly as 4 to 3, can be demon-

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that in moſt animals the ſclerotica are ſo hard as to be incapable of changing the figure of the eye.

Deſcartes aſcribed this contraction and elongation to a muſcularity of the cryſtalline, of which he ſuppoſed the ciliary proceſſes to be the tendons; but this muſcularity he never demonſtrated, neither did he conſider ſufficiently the connection with the ciliary proceſſes.

M. de la Hire maintained, that, for viewing objects diſtinctly at different diſtances, no variations are ever made in any part of the eye except in the ſize of the pupil. The experiment from which he draws this concluſion is related by Dr. Smith, who ſhows it alſo to be fallacious, in page 3. of his remarks.

Dr. Pemberton (Diſſert. Phyſico Medica) advanced, that, in order to ſuit the eye to near objects, the cryſtalline contained muſcular fibres within it, by which one of its ſurfaces was flattened, while the other was made convex; but Dr. Jurin demonſtrates in his eſſay, that ſuch a change as this is too great for a ſubſtance of the conſiſtence of the cryſtalline humour to admit of.

Dr. Porterfield ſuppoſed that the ciliary proceſſes drew the cryſtalline forward, and made the cornea more convex. He made alſo an ingenious experiment, in which he clearly demonſtrated that the eye has a power of altering its conformation, ſo as to ſee diſtinctly at different diſtances. Edinburgh Medical Eſſays, Vol. IV. Dr. Jurin has, however, ſhown that a ſufficient motion of this kind requires a very viſible increaſe in the length of the axis of the eye, an increaſe which has never yet been obſerved.

ſtrated to be $1\frac{1}{3}$ inch from the cornea, ſuppoſing the rays nearly parallel, as in an object at a great diſtance. The rays thus refracted by the cornea fall converging on the cryſtalline humour, and tend to a point 1.228 behind it.

The ſine of incidence to that of refraction of the aqueous into the cryſtalline humour being as 13 to 12, the focal diſtance after refraction will be found to be 1.02 inch from the fore part of it. The rays, therefore, paſs from the cryſtalline to the vitreous humour in a converging ſtate; and the ſines of incidence and refraction being here as 12 to 13, the focal diſtance can be ſhown to be $\frac{6}{10}$ of an inch from the back part of the cornea, which is found to be nearly the

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Dr. Jurin's opinion is, that the uvea is furniſhed with a narrow ring of circular muſcular fibres on the edge next the pupil, and is alſo connected with the cornea, by means of another ring of fibres, which he calls the Great Muſcular Ring, the other being called the Leſſer Muſcular Ring. He therefore ſuppoſes, that, when the eye views objects at the diſtance of 15 or 16 inches, it is perfectly at reſt, no force or effort being uſed by any of its parts; but when objects are viewed nearer, that then the great muſcular ring of the uvea contracts, and thereby makes the cornea more convex. When objects, again, are ſeen at greater diſtances, he ſuppoſes the ligamentum ciliare to contract its longitudinal fibres, and by that means to draw the part of the anterior ſurface of the capſula (which contains the cryſtalline humour, as alſo a water which lies between them), into which theſe fibres are inſerted, a little forwards and outwards. And he then proceeds to ſhow how the cryſtalline humour muſt be a little flattened, by means of the water before it flowing from under the middle, towards the elevated part of the capſula, and the aqueous humour conſequently flowing from above the elevated part of the capſula to the middle; by which means, therefore, the eye can be accommodated to the ſeeing of remote objects as well as thoſe that are near.

This point has been alſo lately the ſubject of a diſpute between Mr. Thomas Young and Dr. Hoſack. Mr. Young (in ſome Obſervations on Viſion in the Phil. Tranſ. 1793), adverting to an obſervation of Dr. Porterfield's, that thoſe who have been couched have not the power of accommodating the eye to difdiſtance of the retina from the cornea; therefore all objects at a great diſtance have their images formed on the retina in the bottom of the eye. But when objects are near the eye, the focal diſtance after the laſt refraction will be a little greater. To make the retina to correſpond to this diſtance, the cryſtalline is moved a little nearer the cornea, by means of the ligamentum ciliare; ſo that in good eyes the cryſtalline can adjuſt itſelf for a due focal diſtance for every diſtance of objects, excepting that which is leſs than 7 or 8 inches.

Definition XVI.

The Axis of the Eye is a line drawn through the

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ferent diſtances, he was led to conclude that the rays of light emitted by objects at a ſmall diſtance, could only be brought to a focus on the retina by a nearer approach of the cryſtalline to a ſpherical form. And he ſays, in examining the eye of an ox, he diſcovered a ſtructure which ſeemed to remove the difficulties that have long embarraſſed this branch of optics. "The cryſtalline of the ox (ſays Mr. Young) was compoſed of various ſimilar coats, each of which conſiſts of ſix muſcles intermixed with a gelatinous ſubſtance, and attached to ſix membraneous tendons. Three of the tendons are anterior, three poſterior. Their length is about two-thirds of the ſemidiameter of the coat; their arrangement is that of three equal and equidiſtant rays, meeting in the axis of the cryſtalline. One of the anterior is directed towards the outer angle of the eye, and one of the poſterior towards the inner angle; ſo that the poſterior are placed oppoſite to the middle of the interſtices of the anterior; and planes paſſing through each of the ſix, and through the axis, would mark on either ſurface ſix regular equidiſtant rays. The muſcular fibres ariſe from both ſides of each tendon. They diverge till they reach the greateſt circumference of the coat; and, having paſſed it, they again converge, till they are attached reſpectively to the ſides of the neareſt tendons of the oppoſite ſurface. The anterior or poſterior portion of the ſix viewed together, exhibits the appearance of three penniformed-radiated muſcles. The anterior tendons of all the coats are ſituated in the ſame planes and the poſterior ones in the continuations of theſe planes beyond the axis. Such an arrangement of fibres can be accounted for on no other ſuppoſition than that middle of the pupil and of the cryſtalline, which conſequently falls upon the middle of the retina.

DEF. XVII. Fig. 15.

The Point of Concourſe is that point C where the two optic axes EC FC meet each other.

DEF. XVIII. Fig. 15.

The Horopter is a ſtraight line AB drawn through the point of concourſe C, parallel to the line EF, which joins the centres of the two eyes or the two pupils.

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of muſcularity. This maſs is encloſed in a ſtrong membraneous capſule, to which it is looſely connected by minute veſſels and nerves; and the connection is more obſervable near its greateſt circumference. Between the maſs and its capſule is found a conſiderable quantity of an aqueous fluid, the liquid of the cryſtalline."

Mr. Young proceeds next to enquire whether theſe fibres are capable of producing a ſufficient alteration in the form of the lens, to account for the known effects, and he finds, that, ſuppoſing the cryſtalline to aſſume a ſpherical form, its diameter will be 642 thouſandths of an inch, and its focal diſtance in the eye .926; and as the human cryſtalline reſembles that of the ox, he ſuppoſes the action of both organs to depend on the ſame principles.

Dr. Hoſack, however, ſtrongly oppoſes this theory of Mr. Young's (Phil. Tranſ. 1794, Part II. page 196.); and he denies the exiſtence of the muſcles which Mr. Young has deſcribed, for ſeveral reaſons, but particularly from the tranſparency they muſt poſſeſs, otherwiſe there would be ſome irregularity in the refraction of thoſe rays which paſs through the ſeveral parts, differing both in ſhape and denſity. He ſays alſo, that, with the aſſiſtance of the beſt glaſſes, and with the greateſt attention, he could not diſcover the ſtructure of the cryſtalline deſcribed by Mr. Young, but found it to be perfectly tranſparent; and he concludes, that no ſuch muſcles as Mr. Young has deſcribed exiſt; and that he muſt have been deceived by ſome appearances that reſembled muſcles. Dr. Hoſack then explains the ſtructure and uſe of the external muſcles of the eye, which are ſix in number; four, called recti, or ſtraight, and two oblique; and by means of theſe, he thinks, the buſineſs is effected.

Def. XIX. Fig. 15.

The Plane of the Horopter is a plane paſſing through AB, and perpendicular to the plane which paſſes through EC and FC.

Def. XX. Fig. 16.

The Common Ray is a line CG drawn from the point of concourſe C and biſecting EF.

Prop. V. Fig. 14.

Every object has its picture painted upon the retina in an inverted poſition.

Let OP be an object, whoſe picture upon the retina is op; therefore, ſince the rays proceeding from the extremities of OP croſs each other in entering the pupil of the eye, it may be ſhown, in the ſame manner as in Prop. IV., that the picture of O, the higheſt part of the object, will be the loweſt part upon the retina,

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And he next proceeds to ſhow how theſe external muſcles are capable of producing the neceſſary changes in the eye, the parts which he ſays are compreſſed by means of the four recti and its axis conſequently lengthened while the oblique muſcles ſerve to keep the eye in its proper ſituation. The convexity of the cornea is alſo increaſed in proportion to the degree of preſſure, and thus the rays of light paſſing through it are neceſſarily more converged; therefore, according to this opinion, we muſt poſſeſs the ſame power of relaxing them in proportion to the greater diſtance of the objects, till we arrive at the utmoſt extent of viſion.

The ſame objections, however, which Dr. Jurin has urged againſt Kepler's theory, may be advanced againſt this hypotheſis of Dr. Hoſack's. I therefore ſhould be more inclined to follow the opinion of Dr. Jurin upon this ſubject than that of any other; for it not only eaſily accounts for the different effects in viſion, but there are alſo fewer objections to it than to the reſt, neither has there been any one who has paid more attention to the ſubject of viſion, or treated it in ſo accurate and philoſophical a manner, as Dr. Jurin.

and that p, the image of the loweſt extremity of the object, will be the higheſt part in the retina. Therefore every object muſt have its picture painted upon the retina in an inverted poſition. Q. E. D. *

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* This propoſition, which was demonſtrated geometrically by Kepler, was firſt confirmed by an experiment exhibited at Rome in 1625, by Scheiner; who, by cutting away the coats of the back part of the eyes of ſheep and oxen, and preſenting ſeveral objects before them, within the uſual diſtance of viſion, he ſaw their images diſtinctly, and beautifully painted upon the naked retina.

This being therefore fully aſcertained, it became a perplexing queſtion with philoſophers, by what means we ſaw objects erect, whoſe images were inverted upon the retina.

Kepler had aſſigned as a reaſon, that as the rays from different points of the objects croſs each other, before they fall upon the retina, we naturally imagine that the impulſe which we feel upon the lower part of the retina, comes from above; and that the impulſe which we feel upon the higher part, comes from below.

Deſcartes, who was of the ſame opinion, illuſtrates this ſolution, by ſuppoſing a blind man to hold in his hands two ſticks croſſing each other, and to puſh the top and bottom of an upright object with their extremities; and ſays, that this man will judge that to be the upper part of the object which he puſhes with the ſtick held in the lower hand, and that to be the lower part of the object which he touches with the ſtick in his upper hand.

Biſhop Berkely however rejected this ſolution, and has given another in its ſtead; in which he has been followed by Dr. Smith, and ſeveral other ingenious writers upon this ſubject. He ſuppoſes that a man born blind, and made to ſee, would not be able at the firſt ſight of an object, without handling it, to form any idea whatſoever of its figure, nor to think any thing he ſaw was high or low, erect or inverted. For, according to his opinion, our ideas of ſight being quite different from thoſe of touch, the notions we form of an object by theſe different ſenſes, will have no ſimilitude; and therefore we can only learn by experience, how one ſenſe will be affected by what in a certain manner affects the other. Therefore, as we find a circle and a ſquare affect the ſight in two different ways, we diſtinguiſh they are different; but which the circle, and which the ſquare, it will be impoſſible to determine, before we have examined them by means of our other ſenſes; and when we have, our eyes alone will always be ſufficient afterwards to know figures of that kind. And, in the ſame manner, as an erect object affects the eye in one way, and the ſame object inverted, affects it in another, we learn to judge, as the eye is affected, whether

Prop. VI. Fig. 16.

If the common ray be at right angles to the horopter, the optic axes will be equally inclined to the horopter.

Let AB be the horopter, CG the common ray, and EC, CF, the optic axes; then, becauſe CG is at right angles to AB, it is alſo at right angles to its parallel EF, and EG is equal to GF; therefore the angle ECG is equal to the angle FCG, but the right angle BCG is equal to the right angle ACG; therefore the remaining angles ACF BCE are equal to each other. Wherefore, &c. Q. E. D.

Cor. If the common ray falls upon the horopter obliquely, the angles which the horopter makes with the optic axes will be unequal. Fig. 17.

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the object is erect or inverted; ſo that, if the pictures upon the retina were erect, in place of being inverted, they would ſhow us, in the ſame way as they do now, thoſe objects when erect, and when inverted.

This, no doubt, would be the caſe, provided the firſt part of the hypotheſis was true, that a blind man made to ſee, could not, by means of his ſight alone, diſtinguiſh the figure of objects, as a circle from a ſquare, or a ſphere from a cube; which laſt was the noted queſtion firſt propoſed by Mr. Molyneux to Mr. Locke, and determined in the ſame way by both of them: But that the contrary would happen, and that the blind man would be able to diſtinguiſh the figures of the two ſolids, before he touched them, I think is ſufficiently demonſtrated by Dr. Jurin (page 28th of the Remarks in Dr. Smith's Optics), contrary to the opinion of theſe two philoſophers.

Therefore, this being granted, let a rod, with a cube fixed on one end, and a ſphere on the other, be placed horizontally before a blind man, ſo as he might feel the cube with his right hand, and the ſphere with his left; ſuppoſe then this man, his hands being removed from the rod, to be made to ſee, he would be able, in the manner deſcribed by Dr. Jurin, to tell the end with the cube, and the end with the ſphere; but the image of the rod being naturally inverted upon the retina, he would ſee the cube towards his left, and the ſphere towards his right, contrary to what he had experienced in feeling them. Therefore, in

PROP. VII.

Every object ſeen by the eye has either its real or apparent place in the line of the horopter or its plane.

This, it is true, cannot be demonſtrated by the ſtrict rules of geometry; but the fact is known by experience, that if an object be ſituated in the point of concourſe, then it appears both in its real place, and is ſeen moſt diſtinctly; whereas, if it be placed on either ſide of that point, it ſtill retains its real place, but appears more obſcure; and the ſame will happen if an object be placed any where in the plane of the horopter; ſo that the nearer an object in the plane is to the point of concourſe, the more diſtinct will its appearance be. If, again, the object be placed in the plane of the optic axes, but out of the horopter, it is found that its ſituation will be changed, and that two appearances of it will be formed ſomewhere in the horopter; but if the real place of the object be neither

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the ſame manner, all other objects would appear inverted to him, if he received his ideas of them directly from their pictures painted upon the retina. And although he might learn by experience and habit to judge of objects as they really were, yet at particular times he would be apt to commit miſtakes, to ſee objects inverted, as when walking in the dark, or half aſleep, or when drunk. This we find, however, is not the caſe; and although upon theſe occaſions we are often deceived with reſpect to the number, poſition, magnitude, and diſtance of objects, imagining ſome to be double in place of ſingle, others appearing large when ſmall, and near when afar off, yet we never ſee them inverted, which, I cannot help thinking, would ſometimes happen, if we formed our ideas of ſight directly from the pictures upon our retina. Wherefore, as I think it appears that at firſt, without touching objects, we could diſtinguiſh thoſe that are erect and thoſe inverted, I ſhould be inclined to ſuppoſe, that, although the retina is a principal inſtrument of viſion, we have ſome other organ connected with it which is no leſs ſo, the impreſſions upon which from the retina give us the ideas of ſight; but what this organ is, and how and in what manner the impreſſions are made upon it, are things of which we are entirely ignorant.

in the horopter nor in the plane of the optic axes, then the apparent place of it will be ſituated, not in the horopter, but in its plane. This being, therefore, found to be univerſally the caſe, and the only principle upon which many of the fallacies in viſion can be accounted for, we may regard this propoſition as one of thoſe laws in nature, the truth of which all experiments confirm, although the cauſe cannot be demonſtrated. Therefore, &c. Q. E. D. *

Cor. I. The apparent place of any object is ſituated in the plane of the horopter, where the ray paſſing through the centre of the eye and the object cuts the plane of the horopter.

Cor. II. The apparent place of any object ſituated in the plane of the optic axes is in that part of the line of the horopter where a ray paſſing through the centre of the eye and that object meets the horopter.

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* That we cannot ſee objects at different diſtances from the eye at the ſame inſtant, and in their true place, is not at firſt ſufficiently obvious; for, in looking forwards to view different objects, we, through habit, are continually changing the angle of the optic axes without perceiving it, and therefore imagine we look at all the different objects before us at the ſame inſtant of time, when in reality we can only ſee thoſe ſituated in the plane of the horopter. A ſingular accident of this kind, that happened ſome years ago, firſt ſuggeſted to me the properties of this line, which until then I was unacquainted with. In travelling one night alone between Thun and Lucerne in Switzerland, I departed from the high road, and took another, to which I was directed, as a ſhorter. The village where I intended ſtaying all night lay towards the left. I therefore inquired at a peaſant to inform me at what part of the road I ſhould turn off. He directed me firſt to paſs one fir tree, and afterwards to walk three miles, until I ſaw three trees ſtanding together a little way before me, then that I ſhould go to my left hand. I therefore went forwards, always looking when the firſt tree was to appear. When I got up to a little eminence upon the road, I imagined I ſaw the firſt; but, walking on a little farther, I diſcovered, in place of one, there were three. I immediately

Prop. VIII. Fig. 19.

Any object ſeen between the horopter and the eye appears double, and its poſition changed with reſpect to the eyes; that is, the right eye ſees the object ſituated towards the left, and the left eye ſees it placed towards the right.

Let O be the object which is ſituated between AB the horopter, and E, F, the centres of the eyes; join EO, FO, and produce them until they meet the horopter in G and H; therefore (Cor. I. Prop. VII.) the object O appears to the right eye F ſituated at G towards the left, and to the left eye E it appears placed at H towards the right. Therefore, &c. Q. E. D.

Cor. If the object lies beyond the horopter, it will alſo appear double; but the apparent places of it will retain their proper ſituation with regard to the eyes, as in fig. 18.

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concluded I muſt have paſſed the firſt without having obſerved it, and thoſe were the three trees from which before I came to I was to turn off. Being ſatisfied of this, I left the road, and went to the left hand, until I came down upon a ſmall lake, where nothing like a houſe or village appeared. I then perceiving my error, returned back, and with ſome difficulty regained my former place upon the road, where, to my aſtoniſhment, in place of three trees only one appeared. I ſtood at firſt a little confounded how this miſtake could have happened; when, by chance, putting up my hand to my face, I diſcovered the ſtring which tied my cap hanging before my eyes. This I ſaw immediately was the cauſe of the whole deception. But how one ſtring came to form the appearance of two, I did not immediately diſcover. From whence it is evident that I had looked before, without changing the angle of my optic axes which were directed towards the tree, and therefore ſaw the ſtring hanging before my face, not only at the ſame diſtance from me as the tree, but double, which conſequently gave the appearancc of the three trees.

Prop. IX. Fig. 20.

If two objects are ſeen in the optic axes between the horopter and the eye, they will have the appearance of three; of which one will appear to be in the point of concourſe, and the other two on each ſide of it.

Let O and P be the places of the two objects in the optic axes EC, FC; join EP, FO, and produce theſe lines until they meet the horopter in G and H; then, as before, the objects O and P will appear to both eyes placed at C, the point of concourſe; but there will alſo be an appearance of it at G, to the eye F, and another at H, to the eye E. Wherefore, &c. Q. E. D.

Cor. The ſame will happen if two objects have their places in the optic axes beyond the horopter, as in fig. 21.

Prop. X. Fig. 22.

If two objects be ſituated out of the plane of the horopter and the plane of the optic axes, then two images of each of thoſe objects will appear in the plane of the horopter.

Let O and P be the two objects neither in the plane of the horopter nor in the plane of the optic axes; join EO, EP, FO, FP, and produce them until they meet the plane of the horopter in K, L, G, and H; therefore (Prop. VIII.) K and L are the apparent places of O and P from the eye E, and G and H their apparent places from F. Wherefore, &c. Q. E. D.

Prop. XI. Fig. 23.

An object appears greater or leſs according as it is ſeen under a greater or leſſer angle.

Let AEa be the axis of the eye, AB, CD, two objects equal in height, ſubtended by two different angles BEA, DEC, of which BEA is the greateſt; produce BE and DE to the retina in b and d; therefore ab, ad, are the pictures or appearances of the objects AB and CD upon the retina, but ab is greater than ad; therefore AB, which is ſeen under the larger angle, appears greater than CD, which is ſeen under the leſs. Q. E. D.

Prop. XII. Fig. 24.

The ſame object appears greater when placed near the eye than it does at a diſtance.

Let AB, CD, be two equal objects placed in the ſame line, at the diſtances EF and EG from the eye E, then AB, the neareſt, will appear greater than CD, which is more remote. Join EB, EA, ED, EC; therefore, if the angle AEB does not contain the angle DEC within it, the ſides EB and EA will either coincide with ED, EC, or lie within them. Suppoſe firſt they coincide with them in H and K; therefore, ſince HK is parallel to DC, HK : DC : : EF : EG, wherefore HK is leſs than DC, but by the hypotheſis it is alſo equal to it; therefore EB, EA, cannot coincide with ED, EC, and much leſs can they fall within them; wherefore the angle AEB is greater than CED; therefore (Prop. XI.) AB appears greater to the eye than CD. Q. E. D.

PROP. XIII. Fig. 25.

The apparent linear magnitudes of objects have a leſs ratio to each other than the reciprocal ratio of their diſtances.

Let the angles ACB, AEB, be the apparent magnitudes of two objects at the diſtances CF and EF from the eye; ſuppoſe alſo the angle ACB is double AEB; therefore the angle CEB=CBE, and CE=CB; but becauſe CFB is a right angle, CB is greater than CF; therefore EC is greater than CF, and EF greater than 2CF; but by hypotheſis the angle ACB has to AEB the ratio of 2CF to CF; therefore the ratio of ACB to AEB is leſs than that of EF to CF; that is, the apparent linear magnitudes of the objects which theſe angles repreſent have a leſs ratio than their diſtances CF and EF. Q. E. D.

Cor. I. The apparent linear magnitudes of objects at great diſtances are reciprocally as the diſtances for the angle ACB, being then very ſmall, CB may be conſidered as equal to CF.

Cor. II. The apparent ſurfaces of objects at great diſtances are reciprocally as the ſquares of the diſtances.

For, ſince the apparent heights and breadths are reciprocally as the diſtances, the apparent ſurfaces compounded of theſe will be reciprocally as the ſquares of the diſtances.

PROP. XIV. Fig. 25.

The diſtances of objects from the eye are as the cotangents of half their viſual angles.

For, ſince EFB is a right angle, if BF be made radius, CF and EF are the tangents of the angles ABC, ABC; that is, the cotangents of FCB, FEB, half of the viſual angles ACB, AEB. Therefore, &c. Q. E. D.

Cor. I. The diſtance of an object from the eye, as alſo its apparent magnitude, being given, its real magnitude may be found.

Cor. II. The diſtance of an object and its real magnitude being given, its apparent magnitude may be known.

Cor. III. The real magnitude of an object and its apparent magnitude being given, its diſtance may be known

Prop. XV. Fig. 26.

The greateſt angle under which any object can be ſeen directly from the eye is leſs than a right angle.

For, let AB be a line of an infinite length, terminated at A, and unlimited towards B, EA a ray of light perpendicular to AB, which enters the eye obliquely; in AB take any point, as D; join ED; therefore, ſince EAD is a right angle, AED is leſs than a right angle; and the greater AD is taken, the nearer will the angle AED be to a right angle, but it will never be equal to one; therefore the greateſt viſual angle of any object which is ſeen directly from the eye is leſs than a right angle. Wherefore, &c. Q. E. D.

Cor. If EF the diſtance of an object AB be equal to one half its length HK, the whole of AB may be comprehended by the eye, but no more, for then the angle AEB is a right angle. Fig. 25.

Prop. XVI. Fig. 27.

From every part of a luminous object the rays of light diverge and flow every where in ſtraight lines through the ſame medium.

For, if the rays did not proceed from every part of the object, ſuch parts as had none flowing from them would be inviſible, ſince nothing can be ſeen without light, either innate or reflected. And they flow in every direction, becauſe in any ſituation of the eye the ſame part of the object appears, and may be ſeen. They alſo proceed in ſtraight lines through the ſame medium; for no reaſon can be given why they ſhould deviate to one ſide more than another. Therefore, &c. Q. E. D.

Cor. I. No rays of light are naturally converging, but always diverge.

For, ſince the rays proceed from bodies, or the parts of bodies, they therefore naturally diverge. When they converge, they are made to do ſo by means of glaſſes, lenſes, &c.

Cor. II. Rays that ſhow things far diſtant fall upon the eye parallel; and, to ſhow things near hand, they diverge from a point.

Cor. III. In order to make an object appear at a great diſtance, the rays flowing from each point of it ſhould be parallel to the eye.

PROP. XVII. Fig. 27.

The denſity of rays proceeding from an object is reciprocally as the ſquare of the diſtance.

Let the rays proceed from the point A, and fall upon the plane BC; then taking the plane BC away, let the ſame light fall on any other plane DE, which may be done by producing the line AB, AC, to D and E. The denſity of the rays upon BC is to the denſity of the rays upon DE as the area of the ſurface DE to the area of the ſurface BC; but the ſurface DE is to the ſurface BC as the ſquare of AD to the ſquare of AB; therefore the denſity of the rays upon BC is to the denſity of the rays upon DE, as the ſquare of AD to the ſquare of AB. Wherefore, &c. Q. E. D.

Cor. I. The brightneſs of an object is the ſame at all diſtances from the eye, if none of the rays be ſtopt by the way.

For the brightneſs is the ſame when the ſame quantity of light falls upon the ſame quantity of the retina in the eye, or when the denſity of light on the retina is the ſame; but the denſity of light falling on the pupil is reciprocally as the ſquare of the diſtance; and the apparent magnitude of the object is alſo reciprocally as the ſquare of the diſtance; therefore the quantity of light in the pupil is as the apparent magnitude of the object, that is, as the ſpace taken up on the retina; therefore the quantity of light being as the ſpace taken up, its denſity is given, and therefore the brightneſs remains the ſame.

Cor. II. Hence the faintneſs of remote objects is owing to the denſity and opacity of the atmoſphere, which ſtops a great part of the light in coming to the eye.

Def. XXI.

Viſion Perfectly Diſtinct, or Perfect Viſion, is that in which the rays of a ſingle pencil are collected into a ſingle phyſical or ſenſible point of the retina.

Def. XXII.

Viſion Imperfectly Diſtinct, or Simply Diſtinct Viſion, is that in which the rays of each pencil are not collected into a ſenſible point, but occupy ſome larger ſpace upon the retina, yet ſo as that the object is diſtinctly perceived.

Prop. XVIII. Fig. 28.

If a circular object be viewed at a diſtance proper for perfect viſion, its picture upon the retina will be circular, and proportional in diameter to the angle which the object ſubtends at the eye; its limb will be well defined, and all parts of the circular picture will be equally ſtrong. But if the ſame circular object be viewed at a diſtance much too ſmall for perfect viſion, its picture upon the retina will be circular, but the diameter will be greater than in proportion to the angle which the object ſubtends at the eye; nor will the picture be equally ſtrong in all its parts, but the middle part will generally be the ſtrongeſt, and will be ſurrounded with a penumbra, growing gradually fainter towards the outer edge, whereby the limb will appear indiſtinct and ill defined.

Let the circle ABDC repreſent that circular ſpace upon the retina which the image of the object would take up, if that image were perfectly diſtinct; or let ABDC repreſent the circular ſpace upon the retina which is occupied by the centres of all the pencils of rays belonging to the indiſtinct image of the circular object, which let be called the true image; alſo, let the circle fghc, having its centre c in the circumference of ABDC, be that circular ſpace upon the retina which is taken up by one of the extreme pencils of rays iſſuing from the object; let this circle be called the circle of diſſipation, becauſe the rays of a pencil, inſtead of being collected into the central point c, are diſſipated all over this circle; alſo, for the ſame reaſon, let its radius be called the Radius of Diſſipation; and let the circle abdfc, concentric with the firſt circle, touch the circle fghe in the point f.

Then that part of the image of the circular object, which is repreſented by the circle abdfc, will be equally ſtrong in all its parts, and will be of the ſame ſtrength as if the image of the object had been perfectly diſtinct.

For (fig. 29.) take any point at pleaſure, as c, within the circle abdC, from that point, as a centre with the radius of diſſipation; deſcribe the circle fghc; then it is evident, as the point c beſtows a part of its light upon every other point of the circle fghc, and receives an equal portion of light from each of thoſe points; ſo that it is juſt as much illuminated as if the rays of its pencil had never been diſſipated, but had been all collected into that one point c, as they would have been, if the image had been perfectly diſtinct; and as this is true of every point within the circle abdC, it is manifeſt that this whole circle muſt be as ſtrongly illuminated as if the image had been perfectly diſtinct, and muſt be equally ſtrong in all its parts.

This part of the true image, repreſented by the circle cabd, which loſes none of its light by diſſipation, but is as ſtrongly illuminated as if it were ſeen by perfect viſion, and is equally luminous in all parts of it, may be called the Falſe Image.

In the ſecond place, the circular ring ABDdba, comprehended between the circumference of the two circles ABD, abd, whoſe breadth is equal to the radius of diſſipation, or that part of the true image ABDC which lies without the falſe image abdc, will not be ſo ſtrongly illuminated as the falſe image abdc, and will gradually grow fainter towards its extremity.

For, let the circles ABDC, abdc, (fig. 30.) repreſent the ſame things as before, and taking two points within the circular ring ABD, dba, one more inward, as c, and the other more outward, as m, from the centres c and m, with the radii cf, mn, each equal to the radius of diſſipation; draw the two circles chfg, mno, cutting the circumference ABD in the points h and f, n and o reſpectively.

Then it is plain that the pencil, whoſe centre is the point c, will diſſipate its rays into the whole circle chfg; but that it will not receive light from every point in that circle, but only from thoſe points of it that are likewiſe ſituated within the circle ABDC.

All thoſe points, therefore, that are comprehended within the lunula hf, beſtow no light on the point c, in return for what they receive from it. The point c, therefore, gives away more light than it receives, and will conſequently appear darker than any point within the circle abdC; and this exceſs of darkneſs will be meaſured by the area of the lunula hf.

In like manner, it will be found that the point m muſt appear darker than any point in the circle abdC, and that this exceſs of darkneſs is meaſured by the lunula no. But the lunula no is greater than the lunula hf; and conſequently the point m, which is nearer to the outſide of the ring, is darker than the point c, which is more inwardly ſituated. The whole ring, therefore, is darker than any part of the circle abdC, and grows gradually darker toward its outer edge. And at the very extremity, it has not half the light of any part of the circle abdC. Alſo, beſides the ring laſt deſcribed, which is darker than the falſe image, or the circle abdC, there is another ring of equal breadth, ſituated without the true image, or the circle ABDC, which is ſtill darker, and whoſe light gradually diminiſhes towards the outſide, till it becomes inſenſible, and at laſt vaniſhes into nothing.

For, let ABDC (fig. 31.) repreſent the ſame thing, as in fig. 28.; and from the centre c, taken any where in the circumference ABD, with the radius of diſſipation cg, draw the circle of diſſipation c*g*hf; and, from the centre C, deſcribe the circle GFHC, touching the circle of diſſipation in the point *g*: Then, between the two circumferences ABD, and GFH, will be comprehended a new ring ABDHFG, of the ſame breadth as the former ABD, dba, fig. 28, 29, 30. This new ring will receive light from the pencil, whoſe centres lie within the former ring; but it will be darker than any part of the former, and its light towards the outer edge, will gradually diminiſh, till it vaniſhes into nothing. From which it follows, that if a circular object be viewed at a diſtance too ſmall for perfect viſion, its picture, upon the retina, will be circular, but the diameter will be greater than in proportion to the angle which the object ſubtends at the eye; and that the middle part will be the ſtrongeſt, the reſt growing gradually fainter towards the outer edge; whereby the limb will appear indiſtinct, and ill defined. Therefore, &c. Q. E. D.

Cor. I. If CA, the radius of the circle ABDC, or of the true image be called r, and the radius of the circle of diſſipation cfgh, be called q; the radius of the falſe image, or of the circle abdc, will be the difference of thoſe two radii, or $r - q$; the breadth of each of the rings ABD dba, fig. 28, 29, 30.; and FGHDBA, fig. 31, will be q, and the radius of the whole appearance, or of the circle CFGH, will be the ſum of the radii of the true image, and of the circle of diſſipation, or $r + q$.

Cor. II. If a rectangular object be viewed at a diſtance much too ſmall, or much too great for perfect viſion, its picture upon the retina will ſtill be rectangular, but the length and breadth will be greater than in proportion to the angles ſubtended at the eye, by the length and breadth of the object; nor will the picture be equally ſtrong in all its parts, but the middle part will be the ſtrongeſt, and will be ſurrounded with a penumbra growing gradually fainter towards the outſide; whereby the limb will appear indiſtinct, and ill defined.

CHAP. II.

Of Images formed by the Reflection of Objects, from reflecting plain Surfaces.

PROP. XIX. Fig. 32.

IF rays proceeding from a radiant point R, fall upon a reflecting plain ſurface A B, at the point D; the focus of the reflected rays will be at r, in the line RC r, perpendicular to the reflecting ſurface; ſo that RC will be equal to C r.

Let RD be the incident ray, and ED the reflected ray. Produce ED, until it meets the perpendicular RC, in the point r: Then (Prop. I.) the angle RDC is equal to ED B, and conſequently to CD r; therefore, in the triangles RDC, r DC, all the angles are reſpectively equal, and CD common; wherefore RC is equal to Cr; therefore the reflected ray DE proceeds from r as a focus, in the perpendicular RCr; ſo that RC is equal to Cr. Q. E. D.

Cor. I. The diſtance of the focus r, from the eye at E, is equal to the incident and reflected rays RD and DE, taken together.

Cor. II. If rays proceeding from an object R, be reflected by the plane AB, to the eye at E, the object will appear to be placed at r, the focus of the reflected rays, where they laſt diverged from.

Prop. XX. Fig. 33, 34.

Images formed by reflections from a plain ſurface, are ſimilar and equal to the objects; and their parts have the ſame ſituation with reſpect to the backſide of the plane ſurface, as the parts of the objects have with reſpect to its foreſide.

From any number of points, D, E, F, of an object, draw the perpendiculars DA, EC, EB, to the reflecting ſurface ACB, and produce them to the points d, e, f, each as far behind the plain ſurface, as D, E, F, are before it; therefore the points d, e, f, are the focuſes of the rays which iſſue from D, E, F, (Prop. XIX.) and, being in the ſame order with infinite others, will conſtitute an image of the object, every way equal and ſimilar to it, as evidently appears, by conceiving the ſurface of the object, and of its image, divided into correſponding lines, ſuch as DEF, def, by planes, ſuch as DdfF, perpendicular to the reflecting plain. Therefore, &c. Q. E. D.

Prop. XXI. Fig. 35.

Let AB repreſent the ſurface of ſmooth water, BP any object ſtanding perpendicular at the water edge, the reflection of this object ſeen by a ſpectator at E, at any diſtance AB from the object, and at any height AE above the ſurface of the water, is equal in every reſpect to the object itſelf.

For, produce the line PB to p, ſo that BP be equal to Bp; join EB, Ep; then, by Prop. XX., Bp is the reflection of the object BP; and, ſince it ſtands upon the water edge, the whole of Bp is ſeen by the eye at E. Therefore, &c. Q. E. D.

Cor. The apparent magnitude of the image is not ſo great as the apparent magnitude of the object, if the eye be ſituated above the ſurface of the water, for the eye is then at a greater diſtance from the image of the object than from the object itſelf.

Prop. XXII. Fig. 36.

Let AB repreſent the ſurface of water as before, AE the height of the ſpectator's eye, BC the diſtance of the object from the water, then CF, that part of the image which is not ſeen, will be to AE, the height of the ſpectator's eye, as CB, the diſtance of the object from the water edge, to AB, the diſtance of the ſpectator.

For, join EB, and produce it until it meets the line of reflection in F; therefore, on account of the ground CB, it is evident that part of the line of reflection CF is not ſeen, and CF : AE : : CB : AB. Wherefore, &c. Q. E. D.

Cor. Fp, the part of the image which is ſeen, is $=CP-\frac{AE \times CB}{AB}$; therefore, when $\frac{AE \times CB}{AB}$ exceeds CP, no reflection is viſible.

Prop. XXIII. Fig. 37, 38, 39.

When the ſurface of water is any way ruffled or agitated by the wind, the reflections of objects appear to vary in their length.

Let AB (fig. 37.) repreſent the ſurface of water when ſtagnant, and CDFGHK the ſurface of the ſame water agitated, aſcending and deſcending in ſucceſſive waves; then, if an object be reflected from L upon the ſide DF of the wave DFG, its image will appear lengthened; but if the ſame object be reflected from M upon the ſide FG, its image will appear ſhortened.

Firſt, let the object BP be reflected from the ſide DF at L (fig. 38.); produce DF until it meets BP produced; therefore (Prop. XX.) the image of BP will be bp every way equal and ſimilar to BP, and inclined to DF at an angle FDp, equal to the angle FDP; but the beginning of the image is not ſeen; for a part of the object equal to AF or BD, the height of the wave above AB, will not be reflected on account of the intermediate waves; it therefore will have the appearance of having commenced at B, and will appear to an eye at E under the angle BEp, which, being greater than the angle BEP, the image, therefore, will appear lengthened.

Again, ſuppoſe the point P of the object reflected from M, upon the ſide FG (fig. 39.) of the wave DFG, then it may be ſhown, as before, that the image will appear to an eye at E under the angle BEp; but this angle being leſs than the angle BEP, the image will appear ſhortened. Wherefore, if the ſurface of water be any way agitated, the reflections from it will appear to vary. Q. E. D.

Cor. The more the ſurface of water is agitated, or the greater the waves are, the more will the images of objects vary in their length.

CHAP. III.

Of the Shadows of Bodies.

Prop. XXIV.

Every opaque body projects a ſhadow in the ſame direction with the rays of light, or towards the part oppoſite to the light.

For, ſince every opaque body prevents the tranſmiſſion of all the rays of light that fall upon it, there will be a ſhadow upon the other ſide of the body, which is not oppoſed to the light; and this ſhadow, being made up of the ſtraight lines in which the rays of light proceed, will be in the ſame direction with theſe rays. Therefore, &c. Q. E. D.

Cor. According as either the luminary or body changes place, the ſhadow likewiſe changes its place.

Prop. XXV. Fig. 40.

Every opaque body projects as many ſhadows as there are luminaries to enlighten it.

Let A and B be two luminaries, and C an opaque body;

Therefore there will be a ſpace D on the oppoſite ſide of C from A, unenlightened by the rays of light which iſſue from A; and there will be another, as E, in the direction of the rays which iſſue from B unenlightened alſo; and although D receives light from B, yet, receiving none from A, it will not be ſo much enlightened as the adjacent parts which receive light from both the luminaries, and therefore will be a ſhadow: for the ſame reaſon E is alſo a ſhadow; wherefore the opaque body C projects two ſhadows; and, if there were more luminaries, it might be ſhown, in the ſame manner, that C would project as many ſhadows. Therefore, &c. Q. E. D.

PROP. XXVI.

Shadows are deeper in proportion as the light of the luminary is more intenſe.

For, ſince a ſhadow is nothing elſe than the abſence of light, the more intenſe the luminary is, the more light will the unenlightened ſpace be deprived of, and therefore it will appear more obſcure when compared with the contiguous parts; or, which is the ſame thing, the ſhadow will appear deeper. Therefore, &c. Q. E. D.

PROP. XXVII. Fig. 41.

If a luminous ſphere be equal to an opaque one, which it illuminates, the ſhadow projected will be a cylinder.

Let AB be the luminous ſphere, DE the opaque one, and DFEMLK the projected ſhadow;

From A and B, the extremities of the diameter AB of the luminous ſphere, draw AD and BE, touching the opaque one DE; and from any point K in AD produced, draw KLM parallel to the diameter DFE, as alſo FL parallel to DK; therefore, ſince AD and BE, the extreme rays, touch the circle DE, they will be at right angles to the diameter DFE; therefore DFLK is a rectangle; which rectangle, being ſuppoſed to be turned round FL as an axis, will generate a cylinder, which is the ſhadow projected by the opaque ſphere DE. Therefore, &c. Q. E. D.

Cor. I. The ſhadow of the opaque ſphere extends the ſame length as the light of the luminous body.

Cor. II. A ſection of the ſhadow will be equal to a great circle of the opaque ſphere.

Prop. XXVIII. Fig. 42.

If the luminous ſphere be greater than the opaque one, the ſhadow projected will be conical.

Let AB be the luminous ſphere, IN the opaque one, and IHN the projected ſhadow.

Becauſe the ray CH touches the ſpheres AB and NI in C and I, the radii GC and MI are perpendicular to CH; wherefore, ſince by the hypotheſis GC is greater than IM, the ray CH will converge towards the ſtraight line GH, which paſſes through the centres of both the ſpheres; therefore, having drawn the cord IN, the triangle IHP will have the angle at P a right angle; wherefore, if the figure KIH be turned round KH as an axis, KIQ will generate the ſphere which is illuminated, and the triangle IPH, the figure of the projected ſhadow; but the figure deſcribed by the triangle is that of a cone; therefore the ſhadow will be conical. Wherefore, &c. Q. E. D.

Prop. XXIX. Fig. 42.

If the luminous ſphere IN be leſs than the opaque one AB, the ſhadow projected CDSR will be a truncated cone.

Becauſe the ray IR touches the ſpheres AB, and IN, in C and I, the radii GC and IM will be perpendicular to IR; wherefore, ſince GC is greater than IM, the ſtraight line IR diverges from the ſtraight line MT, which paſſes through the centres of the ſpheres; therefore, if RS and CD be drawn parallel to each other, RCYT will be a trapezium, having two parallel ſides; therefore, if the figure TECR be ſuppoſed turned round TE as an axis, the quadrant AEG will deſcribe the illuminated hemiſphere, but the trapezium RCYT a truncated cone, which is the ſhadow projected. Therefore, &c. Q. E. D.

Prop. XXX. Prob. Fig. 43.

The diameter and diſtance of a luminous ſphere, and an opaque one being given, of which the luminous ſphere is the greater, to find the length of the ſhadow, or the axis of the ſhady cone projected.

Let B and C be the centres of the two ſpheres, BA the ſemidiameter of the larger, and DC that of the ſmaller, both perpendicular to the ſide ADG of the conical ſhadow DGF, whoſe axis is CG continued to B; draw DE parallel to the ſame axis; then the two triangles ADE and DGC being ſimilar, AE will be to ED, or BC, as DC to CG; that is, as the difference of the ſemidiameters is to the diſtance of the centres, ſo is the ſemidiameter of the opaque ſphere to the axis of the ſhadow, or the diſtance of its vertex from the ſaid opaque ſphere. Q. E. I.

Prop. XXXI. Prob. Fig. 44.

To find the length of the ſhadow AB projected by an opaque body AC from the ſun, the ſun's altitude being given, as alſo the height of the object AC.

Becauſe the ſun's altitude is known, the angle ABC is given, and AC is given alſo; therefore, as tang. ∠ B : radius : : AC : AB, the length of the ſhadow. Q. E. I.

Cor. I. If the length of the ſhadow AB be given, to find the height AC, it will be as radius : tang. ∠ B : : AB : AC.

Cor. II. If the length of the ſhadow AB, and of the object AC be given, to find the ſun's altitude above the horizon, it will be,

as AB : AC : : radius : tang. ∠ B.

BOOK II.

THE THEORY OF PERSPECTIVE; WITH ITS APPLICATION TO THE DIFFERENT SPECIES OF LINES, SURFACES, AND SOLIDS.

CHAP. I.

Of the Theory of Perſpective.

DEFINITION I.

PERSPECTIVE is the art of delineating viſible objects, upon a plain ſurface, placed commonly perpendicular to the horizon, ſuch as they appear to the eye at a given diſtance and height.

To give an idea of the principles and nature of this art, ſuppoſe a tranſparent plane ABDF, (fig. 45.) as of glaſs, raiſed perpendicular on a horizontal plane, and a ſpectator, whoſe eye at E, is directed to the triangle GHK, if we now conceive the rays GE, HE, KE, in their paſſage through this plane, to leave their traces in g, h, and k, there will appear the triangle ghk; which, as it ſtrikes the eye by the ſame rays, gE, hE, kE, by which the particles of light from the triangle, are tranſmitted to the eye, it will exhibit the true appearance of the triangle GHK, though the object ſhould be taken away, the ſame diſtance and height of the eye being preſerved.

II. Fig. 46.

The Point of Sight is the point E, where the ſpectator's eye is placed, to look at the picture.

III.

The Centre of the Picture is the point C, directly oppoſite to the eye, where a perpendicular from the eye, meets the picture.

This point is alſo often called the point of ſight in the picture.

IV.

The Diſtance of the Picture is the length of the line EC, which is drawn from the eye perpendicular to the picture.

V.

An Original Object, whether it be a ſolid, a plane, a line, or a point, means the real object, whoſe repreſentation is ſought.

VI.

The Geometrical Plane, is a plane parallel to the horizon; in which the viſible objects are ſituated, as BDKI.

VII.

The Horizontal Plane, is a plane paſſing through the eye, parallel to the horizon, as ILGH; and is therefore parallel to the geometrical plane, and perpendicular to the picture.

VIII.

The Vertical Plane, is a plane paſſing through the eye, perpendicular to the picture, and to the horizontal plane.

IX.

The Directing Plane, is a plane paſſing through the eye, parallel to the picture, as ILMN.

X.

The Ground Line is the interſection of the picture, and the geometrical plane, as BD.

XI.

The Horizontal Line is the interſection of the picture, and the horizontal plane, as GH.

XII.

The Vertical Line is the interſection of the picture, and the vertical plane.

XIII.

The Directing Line is the interſection of the directing plane, and the ground plane, as MN.

XIV. Fig. 46, 48, 49.

The Vaniſhing Line of any original plane, OPQR, is the line GH; where a plane drawn through the eye, parallel to that original plane, cuts the picture.

XV. Fig. 46.

The Point of Diſtance, is a point in the horizontal line, as G or H; whoſe diſtance from the centre of the picture is equal to the diſtance of the picture EC.

XVI. Fig. 46, 48, 49.

The Vaniſhing Point of any original line, PR, is the point G; where a line drawn from the eye, parallel to the original line, cuts the picture.

XVII.

The Interſecting Point of any original line, is the point where that line produced, cuts the picture.

XVIII.

The Directing Point of any original line, is the point where that line produced, cuts the directing plane.

Proposition I. Fig. 46.

If two or more planes, OPQR, STVX, are parallel to each other, they will have the ſame vaniſhing line, GH.

For, let AFDB be the picture, E the ſpectator's eye, and OPQR an original object; ſuppoſe the plane ILGH to paſs through the eye E, parallel to the plane OPQR, and to cut the picture in the line GH, this line will be the vaniſhing line of the plane OPQR; but, ſince the plane STVX is parallel to OPQR, it will be parellel to ILGH alſo; therefore the line GH is alſo the vaniſhing line of the plane STVX, and in the ſame manner it may be ſhown to be the vaniſhing line of any other plane parallel to OPQR. Wherefore, &c. Q. E. D.

Prop. II. Fig. 46.

The vaniſhing points, G and H, of lines PR OQ, in any original plane, OPQR, are in the vaniſhing line GH of that plane.

Let EG, EH, be drawn from the eye E, parallel to PR and OQ; then, becauſe the plane ILGH is parallel to OPQR, the lines EG and EH will lie in the plane ILGH, and therefore the point G and H, which are the vaniſhing points of PR and OP, will lie in GH, the vaniſhing line of OPQR. Wherefore, &c. Q. E. D.

Prop. III. Fig. 47.

If an original plane, ABCD, be parallel to the picture FKGH, it can have no vaniſhing line upon the picture.

For ſince ABCD is parallel to the plane FKGH, the plane abcd drawn through the eye E, parallel to ABCD, will alſo be parallel to FKGH, and therefore can produce no vaniſhing line upon it. Wherefore, &c. Q. E. D.

Cor. Any line BD, in the plane ABCD, can have no vaniſhing point upon the picture.

PROP. IV. Fig. 50, 51, 52.

The repreſentation ab, of any original line AB, lies in the line CK, which joins the vaniſhing and interſecting points, C and K, of the original line.

For, let the plane ALEF paſs through the eye E, and the original line AB, and cut the picture in the line CK; this line, therefore, paſſes through the vaniſhing point C, and the interſecting point K, of the line AB; but ſince the viſual rays AE, BE, whoſe interſection with the picture, form the repreſentation ab, lie in the plane ALEF, the points a and b will lie alſo in the line CK, which paſſes through the vaniſhing and interſecting points. Therefore, &c. Q.E.D.

Cor. I. When the original line AB is perpendicular to the picture, as in fig. 50. its repreſentation will be in the line drawn from the centre of the picture, to the interſecting point.

Cor. II. If the original line lies in a plane, perpendicular to the picture, but ſituated obliquely in that plane, as in fig. 51. its vaniſhing point will be in the horizontal line, but on one ſide of the centre, and its repreſentation in the line drawn from that point to the interſecting point. And, in like manner, whatever be the ſituation of the original line, its repreſentation will always be in the line joining its vaniſhing and interſecting points.

Cor. III. The repreſentations of all lines which are parallel to each other, but not parallel to the picture, will tend to the vaniſhing point of thoſe lines; and if the original parallel lines be perpendicular to the picture, their repreſentations will tend towards the centre of the picture.

PROP. V.

The repreſentation of any original rectilineal figure, is alſo a rectilineal figure.

For, ſince the repreſentation of any one of its ſides lies in the ſtraight line joining the vaniſhing and interſecting points of that ſide, its repreſentation will be a ſtraight line; for the ſame reaſon, the repreſentation of any other of the ſides will alſo be a ſtraight line; therefore the repreſentation of the figure will be rectilineal. Q. E. D.

PROP. VI. Fig. 53.

If a ſpectator be placed without a circle, the repreſentation of that circle will either be a circle or an ellipſe.

Let CKML be the original circle, FGHN the plane of the picture, and E the eye of the ſpectator; then, ſince the rays KE, LE, &c., by which the circle is ſeen, form a cone, of which the circle is the baſe, the eye the vertex, and a line CE drawn from the centre of the circle to the eye, the axis, any ſection mkl made by a plane FGHN parallel to the directing plane, will either be a circle or an ellipſe. Therefore, &c. Q. E. D.

Cor. If the plane of the picture be parallel to the circle, or if it cut the cone ſubcontrarily, that is, that the angle Ekl may be equal to the angle ELK; in either of which caſes the ſection will be a circle; but if the cone be cut otherwiſe, the ſection will be an ellipſe. (Prop. 23, 24. B. 1. and Prop. 29. B. 2. Con. Sect.)

Prop. VII. Fig. 54.

If a ſpectator be placed in any point in the circumference of a circle, the repreſentation of that circle will be a parabola.

Let KML be a circle, in the circumference of which at D a ſpectator is placed, whoſe eye is at E; therefore, ſince FGHN, the plane of the picture, is parallel to ED, the ſide of the cone, the ſection FIN made by this plane, which is the repreſentation upon the picture of that part of the circle FLK, will be a parabola. (Prop. 25. B. 1. Con. Sect.) Therefore, &c. Q. E. D.

Prop. VIII. Fig. 55.

If a ſpectator be placed within a circle, the repreſentation of any part of that circle will be an hyperbola.

Let KML be the original circle as before, ED the ſpectator, and GHFN the plane of the picture; therefore, ſince this plane is not parallel to either of the ſides EK or EL of the cone, and would cut the circular baſe if produced ever ſo far below the vertex, FIN, the ſection of the cone, which is the repreſentation of that part of the circle FLN, will therefore be an hyperbola. (Prop. 56. B. 3. Con. Sect.) Wherefore, &c. Q. E. D.

Cor. The repreſentation of any part of the horizon is an hyperbola.

Prop. IX.

If a ſpectator be placed without an ellipſe or a parabola, the repreſentation of that conic ſection will be either a circle or an ellipſe.

When the original conic ſection is an ellipſe, ſince it lies wholly on one ſide of the directing plane, every point in that ellipſe will have a real image: its repreſentation, therefore, will be a figure returning into itſelf; and becauſe it is the ſection of an elliptical cone, it muſt, therefore, be a conic ſection, and conſequently either an ellipſe or a circle.

When the original conic ſection is a parabola, as FEG (fig. 56.), let CD be the directing line, and draw any two lines in the parabola, as LM, NO, parallel to CD; and having biſected them in P and Q, through P and Q draw EK, which will be a diameter of the parabola, to which LM and NO will be double ordinates; and becauſe the image or repreſeniation of EK biſects thoſe of LM and NO, the image of EK will alſo be a diameter of the curve produced upon the picture, to which the images of LM and NO will be double ordinates; and becauſe the image of the infinitely diſtant extremity of EK is at the vaniſhing point of that line, the image of the indefinite diameter EK, will be a finite line in the picture, terminated by the image of E and that vaniſhing point; which vaniſhing point is alſo the vaniſhing point of all other diameters of the original parabola, ſince they are all parallel to each other (Cor. 3. Prop. IV.); and the infinitely diſtant extremities of the indefinite ſides EF and EG of the parabola becoming ultimately parallel to its diameters, the images of thoſe extremities will, therefore, meet at the ſame vaniſhing point; and all other points in the parabola, except thoſe infinitely diſtant extremities, having real images, the figure produced upon the picture, muſt therefore be a figure returning into itſelf, and conſequently either an ellipſe or a circle, which will touch the vaniſhing line in the vaniſhing point of EK, and the image of EK will be a determinate diameter of that figure. Wherefore, &c. Q. E. D.

Prop. X. Fig. 57.

If a ſpectator be placed without an hyperbola, the repreſentation of that conic ſection will be a ſemicircle or a ſemiellipſe.

Let FEG be an hyperbola in the original plane (to which HN and HO are the aſymptotes), neither touching or cutting the directing line CD.

Draw any line, LM, parallel to CD, and having biſected it in P, through P, and H the centre of the hyperbola, draw HEK; therefore, LM will be an ordinate to the diameter HEK produced, but the image of EK biſects the image of LM, in the picture alſo; therefore, the image of EK will be a diameter of the curve produced, and the image of LM an ordinate to the curve; but, becauſe the indefinite ſide, EF, of the hyperbola, ultimately coincides with the aſymptote HN, the image of the infinitely diſtant extremity of the indefinite ſide EF, of the hyperbola, is at the vaniſhing point of the aſymptote HN; and, for the ſame reaſon, the image of the infinitely diſtant extremity of the indefinite ſide EG, of the ſame hyperbola, is at the vaniſhing point of the aſymptote HO; therefore the image, or the repreſentation of the hyperbola, FEG, will be a curve paſſing through the image of E, and terminated at the two vaniſhing points of the aſymptotes; and being a conic ſection, muſt conſequently be either a ſemicircle, or a ſemiellipſe. Wherefore, &c. Q. E. D.

PROP. XI.

If a ſpectator be placed in the curve line of any conic ſection, the repreſentation of that conic ſection is a parabola.

This has been already demonſtrated with reſpect to the circle, and the ſame muſt alſo be true with reſpect to the ellipſe, the parabola, and the hyperbola; for, in each of thoſe caſes, the ſection made by the plane of the picture, or the repreſentation of the original curve, is parallel to the directing plane, which touches the ſide of the cone; produced by the original conic ſection, as is evident from fig. 54. where, in place of the circle KML, any of the other conic ſections may be ſuppoſed to be ſubſtituted. Therefore, &c. Q. E. D.

PROP. XII.

If a ſpectator be placed within any conic ſection, the repreſentation of that conic ſection will be the hyperbola.

This has been already ſhown with reſpect to the circle; but if the conic ſection be the ellipſe, let LM (fig. 58.) be taken as the directing line, cutting the original ellipſe FLMG in two points, L and M; the image of either the parts LGM or LFM muſt form a figure whoſe ſides will be indefinite, the extremities of which ſides will therefore ultimately coincide with the images of the tangents to the ellipſe in L and M; wherefore that figure will be an hyperbola, of which thoſe tangents will be the originals of the aſymptotes.

When the given ſection is a parabola, let NO be the directing line, cutting the parabola FEG (fig. 56.) in two points, N and O; then the images of the indefinite ſides, NF, OG, uniting at the vaniſhing point of the diameter EK, that image will be one continued figure, whoſe ſides will be indefinite, the extremities of which will ultimately coincide with the images of the tangents to the parabola in the points N and O; which figure is, therefore, an hyperbola, to which thoſe tangents are the originals of the aſymptotes; and the continued part NEO of the parabola will form another hyperbola, having the ſame aſymptotes, and conſequently oppoſite to the other; and of theſe hyperbolas, that which is formed by the indefinite ſides NF and OG of the parabola, will touch the vaniſhing line in the vaniſhing point of EK; the other hyperbola will fall altogether on the contrary ſide of that line.

If the given ſection be an hyperbola, let LM, the directing line, cut it in two points, L and M, through which points draw two tangents to the hyperbola, fig. 57.

Then the images of thoſe tangents will be tangents to the image of LEM, and alſo to the images of the parts LF and MG, at the infinitely diſtant extremities of thoſe images, the originals of which are L and M, conſequently the image of the part LEM of the hyperbola FEG will be a complete hyperbola, of which the images of the tangents at L and M are the aſymptotes; and, for the ſame reaſon, the images of the remainder FL and GM of that hyperbola will be two indefinite ſides of an oppoſite hyperbola, having the ſame aſymptotes as the other, but terminated each at a point in the vaniſhing line; the image of the infinitely diſtant extremity of FL being at the vaniſhing point of the original aſymptote HN, and the image of the infinitely diſtant extremity of GM being at the vaniſhing point of the other aſymptote HO. Therefore, &c. Q. E. D.

Cor. If the directing line only cut the parabola in one point, or each of two oppoſite hyperbolas in one point, or only one of the hyperbolas in one point, it may be ſhown, in the ſame manner, that the image upon the picture will be the hyperbola.

Prop. XIII.

The repreſentation of any original ſolid, bounded by rectilineal plain ſurfaces, is a rectilineal figure.

For (Prop. V.) the repreſentation of any one of its ſides is a rectilineal figure; therefore, the repreſentation of all its ſides, or of the ſolid, muſt alſo be a rectilineal figure. Q. E. D.

Lemma. Fig. 60.

If from any point two ſtraight lines be drawn touching a circle, thoſe two lines will be equal to each other.

Let BDCE be the circle, and AB and AC two ſtraight lines touching it in B and C; from A draw ADE, cutting the circle in D and E; therefore (Prop. 36. B. 3. Euclid) the ſquare of AB is equal to the rectangle EA, AD, and the ſquare of AC to the ſame rectangle; therefore the ſquare of AB is equal to the ſquare of AC, and the ſtraight line AB to the ſtraight line AC. Wherefore, &c. Q. E. D.

PROP. XIV. Fig. 60.

The repreſentation of any part of a ſphere, is a circle or an ellipſe.

Let BDCE be the given ſphere, of which F is the centre, and which is ſeen by an eye at A.

From A draw AB and AC, touching the ſphere in B and C; join BC, as alſo FB, FC; then, becauſe AB and AC touch the circle BCE, AB and AC (lemma) are equal to each other; and FB, FC, being the radii of the ſphere, are alſo equal; and the ſide AF is common to the two triangles AFB, AFC; therefore the angle BFG is equal to the angle CFG; and in the triangles BFG, CGF, the two ſides BF, FG, are equal to the two ſides CF, FG; therefore the baſe CG is equal to the baſe BG, and the right angle BGF to the right angle CGF; wherefore, if the triangle ABG be ſuppoſed to be turned round AG as an axis, it will generate a cone, whoſe baſe is CB, which is perpendicular to the axis; therefore CB, the ſection of the ſphere, is a circle; wherefore the image of this circle, which is the image of the portion BDC of the ſphere, will be either a circle or an ellipſe. (Prop. VI.) Wherefore, &c. Q. E. D.

Cor. The repreſentation of the part BDC of the ſphere will be a circle, when the ſection BC is parallel to the plane of the picture, or when it cuts the cone ſubcontrarily; in all other caſes it will be an ellipſe.

Prop. XV. Fig. 61.

The greateſt part of a ſphere, which can be contained by any viſual angle, is leſs than an hemiſphere.

Let BACF be a ſphere, which is ſeen by an eye at E, any portion of it BAC which is contained by the viſual angle BEC is leſs than an hemiſphere; for, by the laſt propoſition, the ſection BC of the ſphere is a circle; therefore, if BAC be an hemiſphere, BC will be a great circle of the ſphere, and BAC a ſemicircle, of which BC is a diameter; therefore the angles EBC, ECB, are right angles, and BE, CE, parallel to each other; but they meet at E, therefore they are not parallel; wherefore the part BAC of the ſphere is leſs than an hemiſphere. Therefore, &c. Q. E. D.

Prop. XVI. Fig. 62.

The nearer a ſphere is to the eye, the leſs is that part of it which is ſeen.

Let ABC be a ſphere, whoſe centre is C, ſeen by one ſpectator at E, and by another at F, in the line EC; the part GH, which is ſeen by the neareſt ſpectator at F, is leſs than the part AB, which is ſeen by the ſpectator at E; for, draw EA and EB, touching the ſphere in A and B, and FG, FH, touching it in G and H; join CA, CB, CG, CH; therefore the angles CAE, CGF, are right angles; but the angle EGC is greater than the angle CGF, and conſequently greater than EAC; wherefore the point G falls within the triangle EAC; therefore the angle FCG is leſs than the angle FCA; for the ſame reaſon, the angle FCH is leſs than the angle FCB; therefore the whole angle GCH is leſs than the whole angle ACB; wherefore the part GH of the ſphere ſeen by the eye at F, is leſs than the part AB ſeen by the eye at E. Therefore, &c. Q. E. D.

PROP. XVII. Fig. 62.

The leſs any part of a ſphere is, which is ſeen, the greater will the apparent magnitude of that part be.

Let the greater part AB of the ſphere be ſeen by the eye at E, and the leſs part GH by the eye at F, the apparent magnitude of GH is greater than the apparent magnitude of AB.

Becauſe the two angles CAE, CBE, are equal to two right angles, the two angles ACB, AEB, together, are equal to two right angles; and, for the ſame reaſon, the two angles GFH, GCH together are equal to two right angles; therefore the two angles GCH, GFH, are equal to the two angles ACB, AEB; but the angle ACB is greater than the angle GCH; therefore the angle GFH is greater than the angle AEB; and conſequently the apparent magnitude of GH, the leſs part of the ſphere, is greater than the apparent magnitude of AB, the larger part. Wherefore, &c. Q. E. D.

Prop. XVIII. Fig. 63.

If BFCDGH be a cylinder, and from the eye at E, a line EA be drawn parallel to its axis, till it cut the plane of its baſe in A; and from A there be drawn two tangents to the baſe, meeting it in B and C; and from E there be drawn two other tangents parallel to the former, meeting the cylinder in D and H; if DC be joined, as alſo BH, thoſe two lines will terminate its viſible part.

Becauſe EA is parallel to the axis, EA and DC are parallel; therefore, if a plane be ſuppoſed to paſs through EA and DC, this plane will touch the cylinder in all the points of DC; and if another plane be ſuppoſed to paſs through EA and BH, it will touch the cylinder in all the points of BH; therefore DC and BH terminate the viſible part of the cylinder. Q. E. D.

Cor. I. Wherever the eye is placed in the line EA, the ſame part of the cylinder will remain viſible.

For all lines drawn from any point in EA to any point of DC or BH, will be in the plane EADC or EABH, and conſequently tangents to the cylinder in ſome point.

Cor. II. All the tangents to the cylinder upon the ſide DC, from any point in EA, touch the cylinder in the ſame ſtraight line; and, in like manner, all the tangents upon the other ſide touch the cylinder alſo in a ſtraight line.

Prop. XIX. Fig. 63.

In a cylinder all the circular parts, BFC, DGH, which are ſeen from any point E, in EA, are equal to each other.

Join FC, FB, GD, GH; becauſe CD, BH, and FG, are ſtraight lines upon the cylinder, they are parallel to each other; and ſince the circles BFC and HGD are parallel, CD, BH, and FG, are alſo equal; therefore the two triangles, BFC and DGH, are equal and ſimilar to each other; therefore BC is equal to DH; but ſegments of circles upon equal ſtraight lines, are equal to each other. Wherefore the circular ſegments BFC and DGH are equal. Q. E. D.

Prop. XX. Fig. 63.

The greateſt portion of the ſuperficies of a cylinder which can be contained by any viſual angle, is not equal to the one half of the whole ſuperficies.

Becauſe BA and CA meet each other, BFC is leſs than a ſemicircle, and DGH is equal to BFC; therefore DGH is alſo leſs than a ſemicircle; wherefore the part, BFCDGH, of the ſuperficies of the cylinder, which is ſeen, is leſs than its one half. Q. E. D.

Prop. XXI. Fig. 64.

The nearer a cylinder is to the eye, the leſs is that part of it, which is ſeen.

Let ABCD be a cylinder, ſeen firſt from F, and then from E; the part of the ſuperficies AMNCQR, ſeen from F, which is nearer, is leſs than AOPCST, that part which is ſeen from E. For it may be ſhown, in the ſame manner, as in Prop. XVI. that the circular ſegment GH, is leſs than the circular ſegment

KL; therefore the two circular ſegments MAN, and QCR, are leſs than the two OAP and SCT; wherefore, that part of the ſuperficies AMNCQR, which is ſeen from F, is leſs than that part AOPCST, which is ſeen from E. Q. E. D.

Prop. XXII. Fig. 64.

The leſs of the ſuperficies of a cylinder, which is ſeen, the greater will the apparent magnitude of that part be.

For it may be ſhown, as in Prop. XVII. that the angle HFG, which ſubtends the ſmaller ſegment GH, is greater than the angle KEL, which ſubtends the greater ſegment; therefore, the apparent magnitude of GH is greater than the apparent magnitude of KL; and, for the ſame reaſon, the apparent magnitudes of MAN and QCR, are greater than thoſe of OAP and SCT; wherefore the apparent magnitude of the leſs part of the ſuperficies AMNCQR, is greater than that of AOPCST, the larger part. Therefore, &c. Q. E. D.

Prop. XXIII. Fig. 65.

Let ABCD be a cone, if, from the eye at E, a line be drawn through A, the vertex of the cone, until it cut the plane of the baſe in F, and from F there be drawn two tangents to the circular baſe, meeting it in B and D; alſo, if from theſe points of contact, two ſides, AB, AD, of the cone be drawn, they will terminate that part of the cone which is ſeen from E.

Becauſe, the three ſtraight lines, AB, BF, and AF, meet each other, they are in the ſame plane; for the ſame reaſon, AD, DF, and AF, are in the ſame plane; and thoſe two planes touch the cone in the two lines AB and AD, and EAF is the interſection of theſe two planes; therefore, if any ſtraight line be drawn towards the cone, in either of thoſe planes, it will touch it in AB, or AD, conſequently no part of the cone can be ſeen from E, beyond theſe two lines. Wherefore ADHB is the viſible part of the cone, from E. Q. E. D.

Cor. I. If EA be parallel to the plane of the baſe, then F being infinitely diſtant, the tangents FD, FB, to the baſe, being parallel to EA, will meet the extremities of a diameter of the baſe; and therefore one half of the cone will only be viſible. If the point F fall beyond the cone, more than one half of it will be ſeen; but if F be on the ſame ſide of the cone with the eye, the viſible part will be leſs than the one half.

Cor. II. If the eye be moved any where in the ſtraight line EA, on the ſame ſide of A, the ſame part of the cone will remain viſible; but, if it be moved on the other ſide of A, towards F, the oppoſite part of the cone, BKD, will be the viſible part.

PROP. XXIV. Fig. 65.

The repreſentation of a cone, ABDC, when the eye is ſituated any where in the line EAF, unleſs at the vertex, is a figure bounded by two ſtraight lines, and a circular or elliptical arch.

For the image of it upon the plane of the picture, is the figure contained by the images of the ſtraight lines AB, AD, and the image of the circular ſegment BHD or BKD, which will either be a circle or an ellipſe, according as it cuts the cone of rays proceeding from BHD or BKD ſubcontrarily or not. Therefore, &c. Q. E. D.

Prop. XXV. Fig. 66.

If the eye be placed in the axis of a cone produced upwards, the repreſentation of the cone will be a circle, or an ellipſe.

Let E be the eye placed in AC, the axis of the cone ABD produced upwards, therefore the image of the cone upon the picture, is the image of the circle BCD, which is either a circle, or an ellipſe, according as it cuts the cone of rays proceeding to the eye, ſubcontrarily or not. Therefore, &c. Q. E. D.

Prop. XXVI. Fig. 66.

If the eye E be placed in one of the ſides, AD of the cone produced, its repreſentation will be a circle or an ellipſe.

For its repreſentation is the image of the circle BCD, which is either a circle or an ellipſe, as in the laſt propoſition. Therefore, &c. Q. E. D.

Prop. XXVII. Fig. 67.

If the eye be placed in the plane of the baſe of the cone, the nearer the eye is to the cone, the leſs will the viſible part be, although the apparent magnitude is greater.

Let the eye be firſt placed at E, in the plane of the baſe BCD, and then at a point nearer the cone, as at F. It may be ſhown in the ſame manner as in Prop. XXI. of the cylinder, that the circular ſegment DBE which is ſeen by the eye at F, is leſs than the circular ſegment FBG, which is ſeen by the eye at E; but it may be ſhown alſo, that the angle DFE is greater than the angle FEG; therefore, the apparent magnitude of the viſible part, when the eye is at F, is greater than the apparent magnitude, when the eye is more remote, as at E. Wherefore, &c. Q. E. D.

Prop. XXVIII. Fig. 66.

If the diſtance of the eye from the plane of the baſe of a cone increaſes, while its diſtance from the ſide of the cone remains the ſame, the viſible part of the cone will increaſe, while its apparent magnitude decreaſes; but if the eye approach the plane of the baſe, the viſible part will decreaſe while the apparent magnitude increaſes.

Let the eye be ſuppoſed to move in the line KHL, parallel to AB the ſide of the cone, when the eye is at K, a greater part of the cone is viſible than when the eye is at H; but the apparent magnitude of that part is leſs.

Join AK, AH, and produce them until they meet the baſe in G and F; therefore becauſe LK is greater than LH, the angle BAG is greater than the angle BAF, wherefore BG is greater than BF; therefore by the laſt propoſition, when the eye is at G, the viſible part of the cone is greater, and the apparent magnitude of it is leſs, than when the eye is at F, but (Prop. XXIII. Cor. II.) the viſible part of the cone when the eye is at K, and the apparent magnitude of it, are the ſame as to the eye at G, and when the eye is at H, the ſame alſo as when the eye is at F. Therefore the viſible part of the cone is greater, and the apparent magnitude of it leſs, when the eye is at K than at H. Wherefore, &c. Q. E. D.

Prop. XXIX. Fig. 59.

The repreſentation ab of any line AB which is parallel to the plane of the picture is to the original line AB, as the diſtance Ed of the repreſentation to the diſtance ED of the original.

Becauſe abd is parallel to AB one of the ſides of the triangle AEB, Ed : ED : : ab : AB. Therefore, &c. Q. E. D.

CHAP. II.

The Method of determining the Perſpective Situation of Lines.

Def. XIX.

The Director of the Eye is the interſection of the vertical with the directing plane, and is taken as the meaſure of the height of the eye, and the point where it cuts the directing line is called the Foot of the Eye's Director, or the Point of Station.

Def. XX.

The Line of Station is the interſection of the vertical with the original plane.

DEF. XXI. Fig. 74.

If a line AB be divided in C and D, ſo that the whole line AB is to either of the extremes AC, as the other extreme BD is to the middle part DC, then the line AB is ſaid to be harmonically divided in the points A, B, C and D.

DEF. XXII. Fig. 74.

If a line AB be harmonically divided in A, B, C and D, and from any point, E, without that line there be drawn four lines EA, EC, ED and EB, theſe four lines produced both ways from E are called Harmonical Lines.

DEF. XXIII. Fig. 75.

If through the points A, C, D and B, four lines be drawn parallel to each other, and making any angle whatſoever with AB, thoſe four lines are called Harmonical Parallels.

PROP. XXX. Prob. Fig. 68.

To draw the perſpective ſituation of a ſtraight line ſituated in the ground plane, and parallel to the ground line, its diſtance from which is given.

Let ABOP be the plane of the picture placed perpendicular to the ground plane, GH the horizontal line, C the centre, CE the eye's diſtance, AB the ground line, FK the given line on the ground plane, the perſpective ſituation of which is required, and whoſe diſtance IN from AB is known.

From C, the centre upon the horizontal line GH, cut off CD equal to CE; draw CI perpendicular to AB, and make IL equal to IN; join DL, and through the point M, where CI and DL interſect each other, draw fk parallel to AB; fk will be the perſpective ſituation of the line FK given in poſition.

For CI and DL interſect each other in the ſame point as CI and the ſtraight line drawn from N to the eye, which point is the perſpective repreſentation of N, and becauſe FK is parallel to AB, fk is the perſpective ſituation of it. Therefore, &c. Q. E. I.

Prop. XXXI. Prob. Fig. 69.

To draw the perſpective ſituation of a ſtraight line on the ground plane, and perpendicular to the ground line.

Let RL be the ſtraight line on the ground plane perpendicular to AB, of which L is the interſecting point; join CL, which will be the perſpective ſituation required.

For ſince RL is perpendicular to AB, C is the vaniſhing point of RL, and L is the interſecting point; therefore (Prop. IV. B. II.) CL is the perſpective ſituation of RL. Wherefore, &c. Q. E. I.

Prop. XXXII. Prob. Fig. 70.

To draw the perſpective reſpreſentation of a ſtraight line upon the ground plane, and oblique to the ground line.

Let RL be the given line, the perſpective ſituation of which is required.

At the point E make the angle CED equal to RLA, and join DL; therefore, ſince D is the vaniſhing point, and L the interſecting point of RL, DL is its perſpective ſituation. Wherefore, &c. Q. E. I.

Prop. XXXIII. Prob. Fig. 71.

To draw the perſpective ſituation of a ſtraight line perpendicular to the ground plane, its diſtance from the ground line being given.

Find the perſpective ſeat K of this line, and from K draw KL perpendicular to AB; KL is therefore the perſpective ſituation of the original. Wherefore, &c. Q. E. I.

Prop. XXXIV. Prob. Fig. 72, 73.

To draw the perſpective ſituation of a ſtraight line oblique to the ground plane.

If the line whoſe perſpective ſituation is required, has an elevation above the ground plane, but no declination from the perpendicular; make CD on the horizontal line equal to CE, and the angle CDF equal to the elevation above the horizon, F is therefore the vaniſhing point of the line required, from which, if a ſtraight line be drawn to the interſecting point, it will be its perſpective ſituation; but if the original line be both oblique to the ground plane, and has a declination from the perpendicular, make the angle CED (fig. 73.) equal to it, as alſo DK equal to CE, and the angle DKL equal to the elevation above the horizon, the point L in which KL meets a perpendicular at D will be the vaniſhing point of the original line; therefore, the perſpective ſituation may be found as before. Wherefore, &c. Q.E. I.

Lemma I. Fig. 74.

If two lines harmonically divided, being laid upon each other, coincide in any three points of diviſion, the fourth points of each will alſo coincide.

Let AB and ab be the two given lines laid upon each other, and firſt let the points A, B and C of the one agree with the points a, b and c of the other, whereby the parts AC and ac, which are both extreme parts, as alſo the whole lines AB and ab do agree; then the points D and d will alſo agree.

Becauſe of the harmonical
diviſion of AB - - AB : AC : : DB : DC
And for the ſame reaſon in the line ab, ab : ac : : db : dc
But AB=ab and AC=ac
therefore - - DB : DC : : db : dc
And by compoſition DB+DC=CB:DC::db+dc=cb:dc
but CB is equal to cb, therefore cb : DC : : cb : dc
Conſequently - - DC=dc
Wherefore the points D and d coincide.

Again, let the points A, C and D agree with the points a, c and d, by which means the extreme parts AC and ac, and the mean parts CD and cd, agree: it muſt be ſhown that the points B and b alſo agree.
Becauſe of the harmonical diviſion
of AB, - - AC : CD : : AB : DB
And for the ſame reaſon in the line ab, ac : cd : : ab : db
But AC=ac, and CD=cd, therefore AB : DB : : ab : db
And by diviſion AB—DB=AD:DB::ab—db=ad : db
But AD=ad, therefore - - ad : DB : : ad : db
Conſequently - - - DB=db

Wherefore the points B and b coincide.

After the ſame manner it may be demonſtrated, that if the points C, D and B agree with the points c, d and b, the points A and a will alſo coincide. Q. E. D.

LEMMA II. Fig. 76.

If a line AB be biſected in C, and from any point E without that line there be drawn three lines EA, EC, and EB cutting AB in A, C and B, and through the ſame point E another line EF be drawn parallel to AB, then the four lines EA, EC, EB and EF, produced on both ſides of the point E, will be harmonical lines.

For, draw any line BF cutting EA, EC, EB and EF in G, D, B and F; through D draw HL parallel to AB, which is therefore biſected in D, AC being equal to CB by ſuppoſition; now, in the ſimilar triangles FEB, DLB, FB : DB : : FE : DL=HD
and in the ſimilar triangles
FEG, GHD, - FE : HD : : FG : GD
wherefore - FB : DB : : FG : GD.
Therefore the line FB is harmonically divided in the points F, B, D and G, and conſequently the lines EA, EC, EB and EF are harmonical lines. Q. E. D.

LEMMA III. Fig. 76.

If four harmonical lines EF, EG, ED and EB, formed by the line FB harmonically divided in the points F, G, D and B, be cut by any other line f b parallel to FB, the line f b will alſo be divided harmonically in the points f, g, d and b.

For the parts f g, g d, d b are reſpectively proportional to the parts FG, GD, and DB, (Lem. 1.) Therefore, &c. Q. E. D.

Lemma IV. Fig. 76.

If four harmonical lines EF, EA, EC and EB, formed by the line fb harmonically divided in f, b, d, g, meet in the point E, then any line HL, drawn parallel to any one of the harmonicals, as EF, will be biſected by the other three in the point D.

For it is evident the line HL muſt cut the three harmonicals EA, EC and EB, ſince none of them are parallel to EF, to which HL is parallel by ſuppoſition. Through the middle point D of HL, draw FB parallel to fb, then FB will be harmonically divided in the points F, G, D and B, (Lem. 3.) Now, in the ſimilar triangles FEG, GHD, FE : HD : : FG : GD, and in the ſimilar triangles FEB, DLB,

FE : DL : : FB : DB ; but becauſe FB is harmonically divided FG : GD : : FB : DB, wherefore
FE : HD : : FE : DL ;

conſequently HD=DL. Therefore HL is biſected by the other three harmonicals in the point E. Q. E. D.

Lemma V. Fig. 76.

If four harmonical lines EA, EC, EB and EF, meeting in E, be cut any where by a line FB, that line will be harmonically divided by them in the points F, G, D and B.

Through any of the diviſions of FB as D, draw HL parallel to one of the harmonicals EF, ſo that the point D may be between H and L, then HL will be biſected in D. (Lem. 4). Now, in the ſimilar triangles FEG, GHD, FG : GD : : FE : HD=DL, and in the ſimilar triangles FEB, DLB.

FB : DB : : FE : DL therefore
TG : GD : : FB : DB wherefore
FB is harmonically divided in the points F, G, D and B.
Q. E. D.

COROLLARY I. Fig. 77.

If in an original line any part AB be taken, and biſected in C, not its directing point, the indefinite image of that line will be harmonically divided by the images of A, B and C, and its vaniſhing point.

Let EdeF repreſent the plane paſſing through the eye and the original line eB in which the part AB is taken and biſected in C, then becauſe AC is equal to CB, the lines EA, EC, EB and Ed, which laſt is always parallel to AB, are harmonical lines by Lemma 2. Therefore the indefinite image Fd, which cuts all the four harmonicals, being parallel to none of them, is harmonically divided in a, b, c and d.

COROLLARY II. Fig. 77.

If in the indefinite image of a line any part ab be taken and biſected in c not its vaniſhing point, the indefinite original of that line will be harmonically divided by the originals of a, b, and c and its directing point.

For ab being biſected in c, and Ee being parallel to it, the lines Ea, Ec, Eb and Ee are harmonical lines, therefore the indefinite original eB, which cuts all theſe four harmonicals, is harmonically divided by them in A, B, C and e.

COROLLARY III.

If either of the points of harmonical diviſion of the original line be its directing point, the indefinite image of that line will be biſected by the images of the other three points; and if either of the points of harmonical diviſion of the indefinite image be its vaniſhing point, the original line will be biſected by the originals of the other three points.

LEMMA VI. Fig. 78.

If a line AD be harmonically divided in A, B, C and D, and any two adjoining parts as AB and BC taken together be biſected in m, then mB : mC :: mC : mD. Take Ad = CD, then by ſuppoſition AB : BC :: AD : CD, and by compoſition AB + BC = AC : BC :: AD + CD = dD : CD; therefore $\frac{1}{2}$AC = mC : BC :: $\frac{1}{2}$dD = mD : CD; therefore by diviſion mC—BC = mB : mC :: mD—CD = mC : mD. Q. E. D.

Cor. BC will be to BD as Bm to BA.

LEMMA VII. Fig. 79.

If from a point I without a circle ADBE, there be drawn two tangents ID, IE touching the circle in D and E, and thoſe points be joined by the chord DE, then if any line IF be drawn from I cutting the circle in K and F and the chord of the tangents DE in H, the line IF will be harmonically divided in the points I, K, H, and F.

From I draw the line IMB paſſing through the centre M, and upon IM as a diameter, deſcribe the circle IDME cutting IF in G, draw MG; then ſince the angle IGM is a right angle, KG is equal to GF. Now, becauſe of the circle IDME

IH:HD::HE:HG, & likewiſe becauſe of the circle ADBE KH:HD::HE:HF;
therefore KH:IH::HG:HF.
But the parts KH and HF of the line IF being biſected in G, the line IF is therefore harmonically divided in I, K, H and F (Cor. Lem. 6.) Q. E. D.

Cor. Hence if a line IF, drawn from I through a circle ADBE cutting it in K and F, be harmonically divided in the points I, K, H and F, the point H, if within the circle, will be a point in the chord of the tangents from I.

For in the line IF, three points I, K and F being determined, and the part IK being taken as an extreme part, there can be no point between I and F but one, which can divide that line harmonically, (Lem. 1.) and therefore it muſt be the point H where it is cut by DE the chord of the tangents from I.

LEMMA VIII. Fig. 80.

If from a point K without a circle ADBE, a line KB be drawn through O the centre of the circle, cutting DE the chord of the tangents from K in C, and another line LK be drawn through K perpendicular to KB, and conſequently parallel to DE; then, if from any point L in LK a line LG be drawn through C cutting the circle in F and G, the line LG will be harmonically divided in the points L, F, C and G.

From K to G draw KG, cutting the circle in H and G, and DE the chord of the tangents from K in N, and draw CH; then KG being harmonically divided in K, H, N and G, (Lemma 7.) CG, CN, CH and CK are harmonical lines: from H draw HF parallel to

DE one of the harmonicals cutting the other three CG, CK and CH in F, I and H, then FH will be biſected by them in I (Lem. 4.); but FH being perpendicular to AO which paſſes through the centre of the circle ADBE, and H being a point in the circumference, HF is therefore biſected by AO in I, and conſequently F is a point in the circumference of the circle, as well as in the harmonical CG. Laſtly, draw KF, and becauſe FH is biſected in I, and FH and LK are parallel, KL, KF, KI and KH are harmonical lines, (Lem. 2.); and conſequently the line LG, which cuts thoſe four harmonicals, is harmonically divided by them in the points L, F, C and G, (Lem. 5.) Q. E. D.

Cor. Hence the chord of the tangents from any point L in the line LK muſt paſs through the ſame point C, and no lines in the circle, except ſuch as paſs through C, can be chords of tangents which will meet in any point of LK.

Becauſe from any point L in LK a line may be drawn through C which will be harmonically divided by that point and the circle, therefore the chord of the tangents from L muſt paſs through C. (Cor. Lem. 7.)

But if the propoſed chord do not paſs through C, and the tangent at either of its extremities be produced till it meet LK in any point L, the tangent at its other extremity cannot paſs through the ſame point L, ſince the chord of the tangents from L muſt paſs through C, and no more than two tangents can be drawn to a circle, which can meet each other in one and the ſame point.

LEMMA IX. Fig. 81.

If from any point K without a circle ADBE a line KB be drawn paſſing through O the centre of the circle, and the point C where that line is cut by the chord of the tangents from K be determined, and through K a line L M be drawn perpendicular to KB; then if any point L be taken in that line, and FG the chord of the tangents from that point be produced till it cut LM in another point M, and upon LM as a diameter a ſemicircle LYM be deſcribed, that ſemicircle will cut KB in a point Y between C and O, which point Y will conſtantly be the ſame wherever the point L is taken in the line LM.

Draw LO and MO; then in the triangles LKO, CNO, FG being perpendicular to LO, (Cor. Lem. 2.) the angles CNO and LKO are right, and the angle NOC common to both triangles; therefore theſe two triangles are ſimilar, and in the triangles CNO, CKM, the angles at N and K are right, and the vertical angles at C equal, therefore theſe two triangles are ſimilar, and conſequently the triangle LKO is ſimilar to the triangle CKM

therefore - - LK : KO : : KC : KM;
but becauſe of the ſemicircle LYM, LK : KY : : KY : KM,
wherefore - - KC : KY : : KY : KO.

But the point C in the line KB being conſtantly the ſame as well as the point O, wherever the point L is taken in the line LM, (Cor. Lem. 8.): therefore the lines KO and KC muſt always continue the ſame, and conſequently ſo will the line KY which is a mean proportional between them, and for the ſame reaſon the point Y will always fall between C and O. Q. E. D.

LEMMA X.

The ſame things being ſuppoſed as before, the line KY is equal to KE the tangent to the circle from the point K.

Becauſe of the ſimilar triangles KCE, KEO
KC : KE : : KE : KO
But KC : KY : : KY : KO (Lemma 9.)
Therefore KY = KE. Q. E. D.

Cor. Hence if KY be made equal to KE, and from any point I in the line ML as a centre with the radius IY, a ſemicircle LYM be deſcribed, cutting LM in any points L and M, then a line FG drawn from M through C, and terminated by the circle in F and G will be the chord of the tangents from L; and a line PR drawn from L through C, and terminated by the circle, will be the chord of tangents from M.

PROP. XXXV. Prob. Fig. 82.

To determine the originals of the axis, or any other conjugate diameters of the ellipſe formed by the image of a circle which does not cut or touch the directing line.

Let X be the original plane, AB the directing line, EF the eye-director, and DGHK a circle in that plane, of which C is its centre.

Through the centre C draw a diameter IL perpendicular to AB cutting it in M; from M draw the tangents MN and MO, and join MO cutting IL in P; take MR in MI equal to the tangent MO, and draw RE, biſect it by the perpendicular ST, cutting AB in T; from T as a centre with the radius TE or TR, draw the ſemicircle AERB, cutting AB in A and B;

laſtly, from A and B through P, draw AK, BD, terminated by the circle in G, K, D and H.

Then GK and DH will be the originals of the conjugate axes of the ellipſis, and P the original of its centre, and all lines drawn from A through the circle, and terminated by it, will be originals of double ordinates to the axis whoſe original is DH, and all lines drawn from B, terminated in like manner by the circle, will be the originals of ordinates to the axis whoſe original is GK, and A and B will be the directing points of thoſe ordinates reſpectively.

Becauſe DH is the chord of the tangents to the circle from A, and GK is the chord of the tangents from B, (Cor. Lem. 10.) therefore AK and BD are harmonically divided by the circle and the point P; (Lem. 7.) and A and B being directing points, the images of DH and GK are therefore biſected by the image of P, (Cor. 3. Lem. 5.) wherefore DH and GK are the originals of two diameters, and P the original of the centre of the ellipſe; and becauſe all lines drawn from A, cutting the circle and the line DH, are harmonically divided by the circle and that line, the images of the parts of thoſe lines that lie within the circle are biſected by the image of DH, (Cor. 3. Lem. 5.) and are therefore double ordinates to the diameter repreſented by DH; and becauſe of the directing point A, the images of all thoſe lines being parallel to each other, and to the image of GK, GK is therefore the original of a diameter of the ellipſe, conjugate to the diameter repreſented by DH; and as the images of all lines drawn from B, and terminated by the circle, are parallel to the image of DH, and biſected by the image of GR, they are therefore double ordinates to the diameter repreſented by GR; laſtly, becauſe of the ſemicircle AECB, the angle AEB is a right angle, and EB and EA being the directors of DH and GK, their images are therefore perpendicular, conſequently DH and GK being the originals of two conjugate diameters, which are perpendicular to each other, and to their reſpective ordinates, DH and GK are the originals of the axes of the ellipſe, and A and B are the directing points of their reſpective ordinates; and if through A and B the tangents to the circle AH, AD, BG, BK be drawn, their images will be tangents to the ellipſe in the extremities of the axis repreſented by GK and DH. Therefore, &c. Q. E. I.

Cor. 1. The originals DH and GK of the axes being found, thence to determine which of them repreſents the tranſverſe axis.

Biſect the angle AEB by the line EV, cutting AB in V, and from V through the extremity D of either of the axes DH, draw VD till it cut GK the original of the other axis in Z; then, if the point Z fall without the circle, DH will be the original of the tranſverſe axis; but if Z fall within the circle, DH will be the original of the ſecond axis.

For the angle AEB being biſected by EV, the image of the triangle DZP may be ſhown to be an iſoſceles triangle, having its ſides correſponding to PD and PZ equal; wherefore, if PG be ſhorter than PZ, its image will be ſhorter than the image of PZ, and conſequently ſhorter than that of PD; wherefore PD is the original of the longer or tranſverſe ſemiaxis.

On the contrary, if Z did fall within the circle, the image of PG would be greater than the image of PZ or PD, and PG would then be the original of the tranſverſe axis.

If inſtead of drawing VD, a line VG were drawn, it would cut PD within the circle, which would ſtill ſhow PD to be the original of the longer ſemiaxis.

Cor. 2. The chord of the tangents to the circle from any point A in the line AB, is always the original of a diameter of the ellipſe; and if a line be drawn through the ſame point A and the point P, it will be the original of a diameter conjugate to the other.

For the chord of the tangents from any point in the line AB always paſſes through P (Cor. Lem. 8.) which is the original of the centre of the ellipſe; and A being the directing point of the ordinates to that diameter, a line drawn through that point and the point P muſt be the original of another diameter of the ellipſe parallel to the ordinates of the firſt, and conſequently conjugate to it.

Cor. 3. The diameter IL of the circle, which is perpendicular to the directing line, is always the original of a diameter of the ellipſe; and NO the chord of the tangents to the circle from M, where the perpendicular diameter IL meets the directing line, is always the original of a diameter of the ellipſe conjugate to the diameter repreſented by IL, and IL is the only diameter of the circle, the image of which can be a diameter of the ellipſe.

For IL paſſing through P, is therefore the original of a diameter of the ellipſe, and NO which paſſes alſo through P is the original of another diameter;

and ſince the image of NO, and of all other lines drawn in the circle parallel to NO, and terminated by the circle, are parallel to each other, and biſected by the image of IL, all ſuch lines are the originals of double ordinates to the diameter repreſented by IL; wherefore the diameter repreſented by NO, which is parallel to theſe ordniates, is conjugate to the diameter repreſented by IL; and it is evident, that no other diameter of the circle beſides IL can paſs through P, and therefore that no other diameter of the circle can be the original of a diameter of the ellipſe.

CASE II.

If the centre of the circle be in the line of ſtation, that is, if M were the foot of the eye's director, then IL and NO would be the originals of the conjugate axes.

For IL and NO are the originals of two conjugate (Cor. 3.) diameters, and their images in this poſition of the circle being perpendicular, they are therefore the originals of the axes. Q. E. D.

Cor. If W be the place of the eye, and it be required to determine which of the lines IL or NO is the original of the tranſverſe axis, take MV on the directing line equal to MW the height of the eye, and from V through O draw a line, which, if it cut IL within the circle, will ſhow IL to be the original of the tranſverſe axis; but if it cut IL without the circle, IL will be the original of the ſhorter axe, (Cor. 1.)

For here NO the original of one of the axes being parallel to the directing line, its imaginary director is a line drawn parallel to it through W, and WM is the director of the other axe; if then the right angle made by theſe two directors be biſected by a line from W, it is evident that line muſt cut AB in V, ſo that WM and MV will be equal.

Case III.

If the centre of the circle be in the line of ſtation, and the height of the eye be equal to MR, the image of the circle will be a circle, that is, the ſection of the viſual cone by the picture will be ſub-contrary.

It has been already ſhewn, that when the centre of the forming circle is in the line of ſtation, the lines IL and NO are the originals of the conjugate axes.

It muſt now be ſhown, that at the height of the eye MR the images of IL and NO are equal, and conſequently, that the curve produced is a circle,

Becauſe of the circle DGHK, PI : PO : : PO : PL,
And alſo - - - MI : MO : : MO : ML,
And becauſe IM is harmonically
divided in M, L, P and I, PI : PL : : MI : ML,
Conſequently - - PI : MI : : PO : MO :
But by the ſuppoſition MO is e-
qual to MR the director of the
line MI, therefore - PO : MR : : PI : MI,

And conſequently the images of PI and PO are equal; but the images of PI and PL being equal, as alſo the images of PO and PN, (Cor. 3. Lem. 5.), the image of IL is therefore equal to the image of NO; and conſequently the curve produced by the image of the circle is a circle. Q. E. D.

Def. XXV.

The line which joins the images of the extremities of an original line paſſing through the directing line, is called the Complement of the Image, and the line which joins the originals of the extremities of the image is called the Complement of the Original.

Prop. XXXVI. Prob. Fig. 83.

To determine the originals of the axis, the ordinates, the parameter, vertex, focus and directrix of the parabola, formed by the image of a given original circle touching the directing line of its plane.

Let ABDE be the original circle touching the directing line FA in A, C the centre of the circle, and IF the eye's director.

From the point of contact A to I, draw AI, and perpendicular to it draw IG cutting the directing line in G; from G as a centre with the radius GA deſcribe an arch cutting the circle in H, and join AH; biſect the angle GIA by the line IL, cutting FA in L, and from L as a centre with the radius LA deſcribe another arch cutting the circle in K, and draw LK meeting HA produced in M, and from G through K and M draw GN and GM; then HA will be the original of the axis, and H that of the vertex, O the original of the focus, KN the original of the parameter of the axis, KO and ON will be the originals of two ordinates to the axis, and G the directing point of thoſe ordinates, and GM will be the original of the directrix of the parabola.

From G through the centre of the circle C, draw GP cutting the circle in R and P; then becauſe GA is a tangent to the circle in A, from the point G and

GH is taken equal to it, GH is alſo a tangent to the circle in H from the ſame point G; HA is therefore the chord of the tangents from G, conſequently the images of KO and ON are equal, as are alſo the images of RS and SP (Cor. 3. Lem. 5. and Lem. 7.), wherefore HA is the original of a diameter of the parabola, to which the images of KO, ON, RS and SP are ordinates: but becauſe of the directing points G and A, the angle GIA being a right angle, the images of KN and RP are perpendicular to the image of HA; wherefore HA being the original of a diameter whoſe ordinates are perpendicular to it, HA is the original of the axis, and H is therefore the original of the vertex of the parabola.

Again, becauſe LA is a tangent to the circle in A from the point L, and LK is taken equal to it, LK is therefore alſo a tangent to the circle in K from the ſame point L; but on account of the directing points G, A and L, the angle GIA being biſected by IL, the image of the triangle KMO is an iſoſceles triangle having its ſides repreſented by KO and OM equal; therefore the images of the angles MKO and KMO are equal, but the angle MKO is that which the tangent LK makes with the original of the ordinate KO, and the angle KMO is that which the ſame tangent makes with the original of the axis AH, and the images of theſe two angles being equal, the ordinate repreſented by KO therefore paſſes through the focus; and conſequently the point O, where KO cuts AH, is the original of the focus, and KN the original of a double ordinate to the axis paſſing through the focus and therefore the original of the meaſure of the parameter of the axis; laſtly, becauſe the images of KO and OM are equal, GM, whoſe image is parallel to that of KO (becauſe of their directing point G), is the original of the directrix of the parabola. Q. E. I.

Cor. 1. All lines, as AD, AH, AQ, drawn in the circle from the point of contact A, are the originals of diameters of the parabola, and each of them is perpendicular to, and biſected by that diameter of the circle which paſſes through the directing point of its proper ordinates.

Cor. 2. The diameter AD of the circle, which is perpendicular to the directing line, is always the original of a diameter of the parabola, and the diameter BE of the circle, which is parallel to the directing line, is the original of a double ordinate to that diameter; but no other diameter of the circle beſides AD can be the original of a diameter of the parabola.

Cor. 3. The image of that half of the original of any diameter of the parabola, which is fartheſt from its directing point A, is equal to its complement, but the image of the other half is indefinite.

Cor. 4. The original AH of the axis of the parabola is not only perpendicular to the diameter RP of the circle, which paſſes through G, but their images are alſo perpendicular.

Cor. 5. The infinitely diſtant extremities of the indefinite ſides of the parabola, formed by the image of the circle, become ultimately parallel to its diameters.

Case II.

If the centre of the circle be in the line of ſtation, that is, if A be the foot of the eye's director, and i the place of the eye, the diameter DA of the circle, which coincides with the line of ſtation, will be the original of the axis of the parabola.

For in this ſituation of the circle, the image of DA will coincide with the vertical line, and the images of all lines drawn in the circle parallel to the directing line, will be perpendicular to the image of DA and biſected by it.

Cor. If the height of the eye were equal to CA, the radius of the original circle, then C would be the original of the focus, and BE the original of the meaſure of the parameter of the axis, and the directrix of the parabola would coincide with the vaniſhing line.

Prop. XXXVII. Prob. Fig. 84.

To determine the originals of the aſymptotes, the centre, the axes, and their ordinates, as alſo the vertices of the oppoſite hyperbolas formed by the image of a given original circle cutting the directing line in two points.

Let AFBG be the original circle, cutting the directing line LT in F and G, and let IK be the eye's director.

Draw FS and GS tangents to the circle in F and G meeting in S, and from I draw IF and IG; biſect the angle FIG by the line IM, cutting the directing line in M, and draw SM cutting the circle in A and B; from I draw IL perpendicular to IM, cutting the directing line in L, and join LS, and from A draw AF and AG cutting LS in C and D.

Then SF and SG will be the originals of the aſymptotes, and S the original of the centre of the oppoſite hyperbolas; SA will be the complement of the original of the one half of the tranſverſe axis, and SB the original of the other half, and A and B the originals of its extremities, or the vertices of the oppoſite ſections; and L will be the directing point of the ordinates to that axis; laſtly, CD will be the original of the ſecond or conjugate axis, and M will be the directing point of its ordinates, and FIG will be equal to the inward angle of the aſymptotes.

Becauſe F and G are directing points, whoſe images are infinitely diſtant, the images of SF and SG, which touch the circle in thoſe points, are therefore tangents to the image of the circle at an infinite diſtance; SF and SG are therefore the originals of the aſymptotes, and the point S, where they croſs, is the original of the centre of the oppoſite hyperbolas; and becauſe FG is the chord of the tangents to the circle from S, any line, as SA, drawn from S, within the angle FSG, will cut the circle in two points, A and B, one before and the other behind the directing line, and will be harmonically divided by thoſe interſections and the line FG; (Lem. 7.) and the interſection M of SA with FG, being conſtantly a directing point, the images of A and B will therefore be at an equal diſtance from the image of S, (Cor. 3. Lem. 5.); but the images of A and B are points in the oppoſite ſections, and S being the image of their centre, SA is therefore the complement of the original of one half of a firſt diameter, and SB is the original of the other half, and A and B the originals of the extremities of that diameter. And becauſe the angle FIG is biſected by IM, to which IL is perpendicular, the line LG is harmonically divided in L, F, M and G (Lem 5.); therefore M is a point in the chord of the tangents to the circle from L, (Cor, Lem. 7.); but FG being the chord of the tangents from S, and L being a point in that chord, the chord of the tangents from L muſt alſo paſs through S; wherefore AB which paſſes through M and S is that chord; conſequently the images of all lines drawn from L, and terminated both ways by the circle, are biſected by the image of AB (Lem. 7. and Cor. 3. Lem. 5.); wherefore all ſuch lines are double ordinates to the diameter repreſented by SBA; but becauſe the angle LIM is right, the images of all lines proceeding from L are perpendicular to the image of SBA, therefore this line repreſenting a diameter whoſe ordinates are perpendicular to it, is the original of the tranſverſe axis, and A and B the originals of the vertices of the oppoſite ſections, and L being the directing point of the ordinates to that axis, it is alſo the directing point of the ſecond or conjugate axis which is parallel to them, wherefore LS is the indefinite original of the ſecond axis; and becauſe of the directing points F and G the images of AF and AG being parallel to the image of SF and SG which are the aſymptotes, and paſſing through the image of A, the vertex of one of the ſections, they therefore cut the ſecond axis in its extremities, wherefore C and D, where AF and AG cut LS, are the originals of the extremities of the ſecond axis. And laſtly, becauſe the images of all lines drawn through M are parallel to the image of AB, therefore M is the directing point of the ordinates to the ſecond axis, theſe being parallel to the firſt axis, and the angle FIG being that which the images of SF and SG make together, incloſing the image of the circle, it is therefore the inward angle of the aſymptotes. Therefore, &c. Q. E. I.

Cor. 1. If inſtead of the vertex A, made uſe of to determine the points C and D by the lines AF and AG, the vertex B had been uſed, and the lines GB and FB had been drawn, theſe two lines would have cut LS in the ſame points, C and D, and thereby would have determined the original of the ſecond axis as before.

For LG and SA being both harmonically divided, as before ſhown, and interſecting in M a point of diviſion common to both, and L and S the ſecond points of diviſion in each from the common point M, being joined by the line LS, the lines AF and GB, which join the other points, muſt meet the line LS in the ſame point C; and for the ſame reaſon the lines AG and FB muſt meet LS in the ſame point D (although that point be here out of reach), ſince thoſe lines do not happen to be parallel.

Cor. 2. It is evident that all lines drawn through S, within the angle FSG, and conſequently QP the perpendicular diameter of the circle, which muſt paſs through S, are the indefinite originals of firſt diameters; and all lines drawn through S, without that angle, are the originals of indefinite ſecond diameters. Thus SXY is the complement of the original of a firſt diameter, and TS of a ſecond diameter, but no diameter of the circle can be the original of a firſt diameter of the hyperbolas; beſides PQ, ſince no other diameter of the circle, can paſs through S.

Case II.

If O the centre of the circle be in the directing line LG, the points F and G being then the extremities of the diameter FOG, the tangents FG and GS will be parallel to each other, and perpendicular to the directing line, ſo that the point S will be infinitely diſtant, which point is therefore repreſented by the vaniſhing point of FS and GS, which is the centre of the vaniſhing line, and alſo the centre of the ſections. And all lines which ought to paſs through S, muſt be drawn parallel to FS or GS, ſince their images muſt all have the ſame vaniſhing point; wherefore the angle FIG being biſected by IM, and IL being drawn perpendicular to it, AB and CD drawn through M and L perpendicular to the directing line, will be the complements of the originals of the tranſverſe and conjugate axes, and L will be the directing point of the ordinates to the tranſverſe axis, and M the directing point of the ordinates to its conjugate; and the points C and D, which are the originals of the extremities of the ſecond axis, will be determined by lines drawn from A and B through F or G as before. Q. E. I.

Cor. All lines drawn through any point between F and G, parallel to AB, are the complements of the originals of firſt diameters, and the tangents to the circle from the extremities of any of the lines, thus drawn other than the diameter PQ, muſt meet in ſome point in the directing line through which point the original of the ſecond diameter conjugate to that firſt diameter muſt paſs, parallel to AB; and the originals of the extremities of ſuch ſecond diameter are found by lines drawn through the extremities of its correſponding firſt diameter and the point F or G.

Case III.

If the centre of the circle be in the line of ſtation, but not in the foot of the eye's director, that is, if N be the point of ſtation, then QP, the diameter of the circle perpendicular to the directing line coinciding with the line of ſtation SPQ, will be the complement of the original of the firſt or tranſverſe axis, ſince the images of all lines drawn in the circle, parallel to the directing line, will be perpendicular to the image of QP, and biſected by it, and the diameter RV, which is parallel to the directing line, will therefore be the original of a double ordinate to the axis ; the original of the conjugate axis muſt therefore paſs through S the original of the centre of the hyperbolas, parallel to the directing line, and its extremities are determined by lines drawn from F and G through Q or P as before.

But if the centre of the circle O be the foot of the eye's director (Fig. 85.), the tangents FS and GS being then parallel, the point S is infinitely diſtant, and therefore the conjugate axis hath no real original, becauſe that original ought to be a line paſſing through the infinitely diſtant point S, parallel to the directing line ; no ſuch line as which can be drawn, however, as the image of this infinitely diſtant point is the vaniſhing point of QP, which is the centre of the vaniſhing line ; the vaniſhing line itſelf muſt be the image of the imaginary original of the ſecond axis, and the extremities of this ſecond axis are found by lines drawn in the picture through the images of Q and P, which are the vertices of the oppoſite hyperbolas parallel to the aſymptotes as before Q. E. I.

But here, as in the preceding caſe, the diameter FG of the circle coinciding with the directing line, it can have no image.

Cor. 1. If the foot of the eye's director, or point of ſtation, fall within the circle, and the height of the eye be taken a mean proportional between the ſegments of the directing line made by the circle, and the point of ſtation, then the oppoſite hyperbolas formed by the image of the circle will be equilateral, that is, the aſymptotes will be perpendicular to each other, and the tranſverſe and conjugate axes will be equal.

For if FG be the directing line, and M be the point of ſtation, and the height of the eye Mj be a mean proportional between FM and MG, the ſegments of the directing line, it is evident, a ſemicircle deſcribed on FG as a diameter, will paſs through j, and therefore that the angle FjG, which is the angle made by the aſymptotes, will be a right angle.

Cor. 2. If the height of the eye be taken greater than a mean proportional between the ſegments of the directing line terminated as before, the inward angle of the aſymptotes will be acute, and the tranſverſe axis will be longer than the conjugate. If the height of the eye be leſs than the mean proportional between the ſegments of the directing line, the inward angle of the aſymptotes will be obtuſe, and the ſecond axis will be longer than the tranſverſe.

Cor. 3. If the point of ſtation be out of the circle, no equilateral hyperbolas can be produced, whatever height of the eye be taken.

For if K be the point of ſtation, it is evident, that the angle FIG muſt be acute, whatever point I in the perpendicular KI be taken for the place of the eye, for if K do not fall ſomewhere between F and G, no point in KI can cut a ſemicircle drawn on FG as a diameter.

CHAP III.

The Doctrine of Perſpective Meaſures, or the Method of dividing Perſpective Lines in any Proportion required.

Prop. XXXVIII. Prob. Fig. 86.

To divide a perſpective line lying parallel to the ground line, and whoſe original is ſituated in the ground plane.

Let ABOP be the plane of the picture, GH the horizontal line, ab the perſpective line given, and c, a given point in it, from which it is required to cut off any particular length, as for example, two diviſions.

From one of the points of diſtance, as D, draw the line Dc to C; ſet off two diviſions from C to d, and then draw Dd meeting ab in e, then ce will be the meaſure required, cut off from c.

For ſince DE is the diſtance of the picture, and ce is parallel to the ground line, ce will have the ſame ratio to its original as it has to Cd; therefore the original of ce is equal to Cd, or the given meaſure required. Wherefore, &c. Q. E. I.

Prop. XXXIX. Prob. Fig. 87.

To divide a perſpective line that is perpendicular to the ground line, and whoſe original is ſituated in the ground plane.

Let ABOP be the plane of the picture as before, D the point of diſtance in the horizontal line, and CE the given perſpective line; let it be required to cut off Em equal to a certain meaſure.

Make Ea equal to the ſaid meaſure, and draw Da cutting CE in m; therefore Em is cut off equal to the meaſure required, for the original of Em is equal to Ea.

If from l in the line Ck, which is another perpendicular to the ground line, it is required to cut off a certain meaſure towards k, draw the line Dl, and continue it till it cuts the ground line, which is here at E; ſet off the meaſure required from E to n, and draw Dn cutting Ck in o; then, for the ſame reaſon as before, l o will be the meaſure required. Wherefore, &c. Q. E. I.

Prop. XL. Prob. Fig. 88.

To divide a perſpective line that is oblique to the ground line, whoſe original is in the ground plane.

Let ABOP be the plane of the picture, C the centre, CE the diſtance of the picture, and FG the given perſpective line, whoſe vaniſhing point is G, and let it be required to cut off from F a certain meaſure, as Fh.

Join EG, and on the horizontal line ſet off GD equal to GE: From F the interſecting point, ſet off alſo FH equal to the meaſure wanted; join DH meeting FG in h; then, ſince GD is equal to the diſtance of the eye from the vaniſhing point, the original of Fh will be equal to FH the meaſure given. Wherefore, &c. Q. E. D.

Prop. XLI. Prob. Fig. 89.

To divide a line perpendicular to the ground plane.

Let ab be the image of a line perpendicular to the ground plane, and let it be required to cut off from the point a in it any number of diviſions or any given meaſure.

Take at random any point G in the horizontal line, and from thence draw a line GC to any point C in the ground line. Set off CD equal to the given meaſure in the perpendicular CE and join DG; after this, draw a line from the foot a of the object parallel to the ground line, cutting GC in the point d, and raiſe the perdendicular de till it meets DG in e, then de will contain the meaſure required. For ſince DG and CG tend to the ſame point in the horizontal line, their originals are parallel to each other; but the original of de is alſo parallel to CD, therefore the original of de is equal to CD, which was made equal to the given meaſure. Therefore, &c. Q. E. I.

Prop. XLII. Prob. Fig. 90.

To divide a perſpective line whoſe original is oblique to the ground plane.

Let the line be DE, and let it be ſuppoſed that a part equal to a given meaſure is required to be cut off from the point D.

From the vaniſhing point E, draw EF perpendicular to the horizontal line, and from F ſet off FG equal to the diſtance of the eye; join EG, and from D draw DH parallel to EG: Upon DH make DK equal to the given meaſure; join GK, cutting DE in the point e, and De will be the part required to be cut off.

For becauſe FG is equal to the diſtance of the eye, and DH is drawn parallel to EG, GK and DE interſect each other in the ſame point as a line drawn from K to the eye, ſuppoſing DK to be an original line. Therefore, &c. Q. E. D.

CHAP. IV.

The Method of Repreſenting Figures that are in Planes Perpendicular to the Picture.

Prop. XLIII. Prob. Fig. 91.

To find the repreſentation of a triangle, one of whoſe ſides is parallel to the picture.

Let DLF be the given triangle, having the ſide DF parallel to the ground line AB.

Produce the two ſides LD, LF, till they meet the ground line in N and Q; from D draw DM parallel to LF; and from E, the point of ſight, draw EH, EK, parallel to the two ſides of the triangle, meeting the horizontal line in H and K; join KN, KM, and draw HQ cutting them in l and d, and from d draw df parallel to AB: Then the triangle dlf is the repreſentation of the triangle DLF.

Becauſe EK and EH are parallel to DL and FL, K and H are the vaniſhing points of FL and DL, and N and Q are their interſecting points; and becauſe DM is parallel to FL, K is the vaniſhing point of DM: Therefore, Prop. IV. Book 2., the triangle dlf is the repreſentation of the triangle DLF. Wherefore, &c. Q. E. I.

Prop. XLIV. Fig. 92.

To find the repreſentation of a triangle, when one of its ſides is perpendicular to the ground line.

Let DFL be the triangle, having the ſide DL perpendicular to AB the ground line; produce the three ſides of the triangle, DF, DL and FL, till they meet the ground line in M, N and Q; find the vaniſhing points K and H of the ſides FL and DF, as before; join CN, HM, and KQ; therefore the triangle dlf, formed by their interſections, may be demonſtrated, as in the laſt, to be the repreſentation of the triangle DLF. Wherefore, &c. Q. E. I.

Prop. XLV. Fig. 93.

To find the repreſentation of a triangle, when its ſides are ſituated obliquely with reſpect to the ground line.

Let the given triangle be LDF, having its three ſides oblique to the ground line AB; find the three vaniſhing points, G, K, and H, and produce the three ſides of the triangle until they meet the ground line in the interſecting points M, N, and Q; draw GQ, HM, and KN: Therefore it may be ſhown, in the ſame manner as before, that the triangle dlf is the repreſentation of the given triangle DLF. Therefore, &c. Q. E. I.

Prop. XLVI. Prob. Fig. 94.

To find the repreſentation of a ſquare, having one of its ſides parallel to the ground line.

Let DFLM be the given ſquare, having the ſide LM parallel to AB; produce the two ſides DL and FM, till they meet AB in Q and R; draw CQ, CR, and alſo the diagonal FL, meeting AB in N; find the vaniſhing point H of the line FN, and draw HN meeting CQ and CR in l and f, and draw lm and fd parallel to AB. The figure dfln is the repreſentation of the ſquare DFLM.

Becauſe LM is parallel to AB, the centre C is the vaniſhing point of DQ and FR; and becauſe H is the vaniſhing point of FN, lf is the image of LF: Therefore, dfln is the image of the ſquare DFLM. Therefore, &c. Q. E. I.

Prop. XLVII. Prob. Fig. 95.

To repreſent a ſquare ſituated obliquely to the ground line.

Let DFLM be the given ſquare; produce the two ſides DF and DL, till they meet the ground line in Q and N; draw alſo the diagonal DM until it meets the ground line in S, and the ſide FM till it meets it in R. Find the vaniſhing point H of the lines DN and F R, as alſo the vaniſhing point K of DS : join HN, HR, and KS, cutting HN and HR in d and m ; draw d Q cutting HR in f, and lS cutting HR in m. The figure df lm may be demonſtrated, in the ſame manner as in the laſt propoſition, to be the repreſentation of DFLM. Therefore, &c. Q. E. I.

Prop. XLVIII. Prob. Fig. 96.

Let DFLM be the given circle ; deſcribe a ſquare DFLM about it ; find the image df lm, as in the laſt propoſition, and within this image inſcribe, as nearly as poſſible, the figure of an ellipſe, which ſhall touch the four ſides of the image : Therefore, Prop. VI. Book 2., that figure will be the image of the given circle. Wherefore, &c. Q. E. I.

Method Second.

Find by Prop. XXXV. the originals of the axes of the ellipſe, formed by the image of the given circle, and then deſcribe the ellipſe, which will be the perſpective repreſentation of the circle.

Note. The repreſentations of figures ſituated in planes not perpendicular to the picture, are found in the ſame manner, by only ſubſtituting, in place of the horizontal line, the vaniſhing line of the given plane.

CHAP. V.

Of Objects that are Perpendicular to the Ground.

Prop. XILX. Prob. Fig. 97.

To find the repreſentation of a plane, having its baſe perpendicular to the bottom of the picture.

Case I.

When one corner D of the given plane DFKL, 10 feet high and 6 feet broad, is at the bottom of the picture, draw DC to the centre, and DL perpendicular to AB.

From D upon DL ſet 10 feet, and upon DC ſet DF equal to 6 feet; join LC, and from F draw FK parallel to DL: Therefore, Prop. XXXIX and XLI. B. 2. DFKL is the image of the plane required.

Case II.

Let it be required to draw the repreſentation of a plane of the ſame height and breadth with the former, but at 5 feet diſtance from the picture.

From C draw any line CD, and from D on DC ſet DM equal to 6 feet, MN equal to the breadth of the plane, and make DL equal to the height of the plane; join LC, and draw MR, NQ parallel to DL: Therefore, for the ſame reaſon as before, MNQR is the repreſentation of the given plane at the required diſtance from the picture. Wherefore, &c. Q. E. I.

PROP. L. Prob. Fig. 98.

To find the repreſentation of a plane parallel to the picture.

Let the given plane be 10 feet high, 8 feet wide, and 6 feet from the bottom of the picture; anywhere to AB from C draw CD, and from D cut off DK equal to 6 feet. From K draw a line parallel to AB, in which find KL equal to 8 feet; draw KM perpendicular to KL, equal to 10, and complete the parallelogram KLMN, which will be the image of the plane required. Therefore, &c. Q. E. I.

PROP. LI. Prob. Fig. 99.

To find the image of a plane, having its baſe oblique with the bottom of the picture.

Let G be the vaniſhing point of the baſe, and K the interſecting point, and let the plane required be 12 feet in height, 6 feet in breadth, and 4 feet from the bottom of the picture.

From K ſet off KA equal to 4 feet, and draw KL perpendicular to the ground line, equal to 10 feet; join LG, and draw CD parallel to LK; cut off CB equal to 6 feet: Therefore, for the ſame reaſon as in the other propoſitions, the figure CBFD is the image of the plane required. Wherefore, &c. Q. E. I.

Prop. LII. Prob. Fig. 101.

To find the repreſentation of triangular pieces of wood, &c., when they are either above or below the horizontal line.

Case I.

When they are below the horizontal line, and have one ſide, a b e d, parallel to the picture.

Find the vaniſhing points H and L, of the oblique ſides; then from b and d, draw bL, dH, cutting each other in c; then is bcd the top, which completes the figure. In this figure, the front ſide abde is parallel to the picture, but in the figure nopqrm, the back ſide npmq is parallel to the picture; therefore let us find the appearance of that alſo: Here let mqpn be the parallel ſide given. Through the points n and m draw lines from H, and through the points p and q draw lines from L, which will cut the lines Hm, Hn, continued in o and r; therefore draw o r, and the thing propoſed is done. Or it may be found by giving one edge mr. Continue mr to its vaniſhing point H, and draw rL; then draw mq parallel to the horizontal line, cutting rL in q; then is rq the other ſide; therefore draw mn, ro, pq, each perpendicular to the horizontal line, and make any of them the propoſed height (ſuppoſe ro), then draw oH, oL, cutting mn, and qp, in n and p, and then draw np, which finiſhes the figure.

Case II.

When they are above the horizontal line.

Let fg be one of the bottom edges given. Draw fi perpendicular to the horizontal line, and make it equal to the propoſed height; from f draw fL, then draw gl parallel to the horizontal line, cutting fL in l; and then is fl the depth of the other ſide; therefore, draw gh and lk parallel to fi, and from i draw iH and iL; which completes the repreſentation.

Prop. LIII. Prob. Fig. 100.

To find the repreſentation of any triangular figure, when all the ſides are oblique to the picture.

Case I.

When it is below the horizontal line.

Let AD be one edge given, whoſe vaniſhing point is out of the picture, and let L be the vaniſhing point of the other edge AG; make AG equal to AD, and from A, D, G, draw lines perpendicular to the ground; then make AB equal to the height, and draw BL, which will complete the ſide ABFG. Again, continue AB and DE to the horizontal line, and make DE to DH as AB is to AR; then draw BE, which completes the ſide ABED. Finally, draw EF, which finiſhes the repreſentation: For if AD and BE were continued, they would both vaniſh into the ſame point in the horizontal line.

CASE II.

When it is above the horizontal line.

Let HO be one edge given, whoſe vaniſhing point is out of the picture, and let L be the vaniſhing point of the edge HN. Make HN equal to HO, as before, and from N, H, O, draw lines perpendicular to the ground; then make HK equal to the propoſed height, and draw KL, which will give one ſide. Again, make PQ the part above the horizontal line to PO the part below the horizontal line, as IK is to IH, and draw KQ, which will complete the repreſentation propoſed: For if HO and KQ were continued, they would meet in a point upon the horizontal line.

PROP. LIV. Prob. Fig. 103.

To find the repreſentations of cubes, both above and below the horizontal line, when ſome of their ſides are parallel to the picture.

CASE I.

When below the horizontal line.

Let abeg be one ſide given; draw bC, eC, and gC, and find the vaniſhing point H of the diagonal ef, and draw eH, cutting Cg in f; then from f draw fd parallel to the horizontal line, cutting eC in d; and then draw dc parallel to eb, and the repreſentation is completed; and for the cube E, make the front ſide like the cube A, and draw lines from the upper corners to the centre C; then, by continuing eg and fd, we may complete the other alſo, as in the figure: And therefore, having got one repreſentation, that will be ſufficient for any number of the ſame kind, provided they ſtand all in the ſame line a n, that is, at the ſame diſtance from the bottom of the picture.

CASE II.

When above the horizontal line, as B and D.

Here it is to be obſerved, that the rule in either caſe is the ſame; and therefore we are to proceed in the ſame manner in finding the repreſentation of a cube above the eye, as we have done in determining the appearance of a cube below the eye, which is ſufficiently explained by the figures. And ſo likewiſe for the depth mo, of the parallelopiped F, which is found by drawing a line from the corner n, to the vaniſhing point of the diagonal.

PROP. LV. Fig. 102.

Of a Cube and Parallelopiped, whoſe ſides are oblique with the Picture.

CASE I. Of the Cube.

Let ab be given, whoſe vaniſhing point is L; and let H be the vaniſhing point of the other ſide ag. From a draw a 1, parallel to the horizontal line, and make LA equal to the diſtance LE; then through b draw A 1, cutting a 1 in 1; and then is a 1 equal to ab; and from a and b draw lines perpendicular to the horizontal line, and make ae equal to a 1; then from e draw eL, cutting bc in c, and then we ſhall have one ſide. Again, from c and e draw lines to H, and from e draw a line to C (the vaniſhing point of the diagonal), which will cut cH in d; then from L draw a line through d, cutting eH in f. Finally, from f draw fg parallel to ae, and then will the repreſentation be completed.

Case II.

Of a parallelopiped, or oblong piece of wood, reſting upon one of its longeſt faces.

Let it be required to make it 3 feet long, 1 foot thick, and 1 foot high; and let o be the neareſt corner, and HL the vaniſhing points of the ſides. Through o draw 2 o 3, parallel to the horizontal line, and ſet off 3 feet upon it; then draw oH and oL, and make HB equal to HE, and draw B2 cutting oH in i; then is oi equal to 3 feet. Again, make LA equal to LE, and from o ſet off o3, equal to 1 foot; then draw 3A, cutting oL in n; and then is o n equal to o3. Again, from i, o, n, draw lines perpendicular to the horizontal line, and make om equal to o3. Finally, from m draw mH and mL, cutting ih in h, and nl in l; then from l draw a line to H, and from h draw a line to L, which will cut each other in k, and ſo finiſh the repreſentation, which will be 3 feet long, 1 foot thick, and 1 foot high.

BOOK III.

THE PRACTICE OF PERSPECTIVE,

WITH REGARD TO THE REFLECTIONS AND SHADOWS OF BODIES,

AND ITS APPLICATION TO THE DRAWING OF ARCHITECTURE.

CHAP. I.

Of the Reflections and Shadows of Bodies.

Prop. I. Fig. 104.

Let AB be an object ſtanding upon an eminence, of which B is its bottom, it is required to draw its reflection.

Produce the ſides of the object downwards at pleaſure, as the pricked lines in the figure, and let C be where the even ground cuts the bottom of the hill, then make CD equal to CA, which will give the length of the reflection, by Prop. XX. B. 1. Therefore, &c. Q. E. I.

Cor. 1. In the object G, which ſtands upon the flat ground, make the length of the reflection equal to the object itſelf.

Cor. 2. In the inclined object H, the reflection muſt have the ſame angle of inclination with the real object, and be of the ſame length.

Prop. II. Prob. Fig. 105.

To draw the ſhadow of a body projected by a lamp or candle.

Let L be the place of the luminary in the plane of the picture, and P the point perpendicularly under it in the ground plane, and ABCDEF the body whoſe ſhadow is required to be drawn.

Draw PD, PE, and PF, and produce them until they meet lines drawn from L through A, B, and C, in G, H, and K; therefore, DGHK will be the ſhadow of the body (Prop. XXIV. B. 1). Wherefore, &c. Q. E. I.

Cor. If the ſhadow of the rod ab, which ſtands obliquely to the ground plane, be required, let fall a perpendicular bc to the ground plane, and draw Lb and Pc until they meet in d; then join ad, which will be the ſhadow of the rod.

Prop. III. Prob. Fig. 106.

To draw the ſhadows of bodies projected by the ſun.

Since ſhadows projected by a lamp or candle are determined by finding the point under the luminary, in the ſame manner the ſhadows from the ſun are drawn by finding that point in the horizontal line, in or over which the ſun muſt always be ſuppoſed to be, by reaſon of his immenſe diſtance.

If, for inſtance (Fig. 106.), the ſun be 30 degrees to the left hand of it, make the angle DEH equal to 30 degrees, and H will be that point in the horizontal line over which the ſun may be ſuppoſed to be perpendicular; and nothing now is required for fixing its preciſe place above the horizon, but to know its altitude: Thus, if the ſun be 35 degrees high, make HI equal to HE; and, raiſing the perpendicular HK, make the angle HIK equal to 35 degrees, and K will be the ſun's place.

If now it is required to draw the ſhadow of any object made by the ſun in this ſituation, I ſuppoſe K to be the place of the candle in Fig. 105, and H to be b, the point upon the ground plane that is perpendicularly under it, and proceed exactly as in the laſt propoſition. Thus, to draw the ſhadow of the ſolid (X), Fig. 107, the place of the ſun being at K, I draw Ha, and interſecting it by Kb in c, I have the place of the ſhadow of b; and joining ac, I have the ſhadow of the perpendicular line ab. In like manner I find d, the ſhadow of f, at the top of the line ef; and alſo g, the ſhadow of the vertex of the line hi. Then, joining theſe points, I have the ſhadow of the whole object (X). Q. E. I.

Note. When, as in this example, the place of the ſun is fixed above the horizontal line, it is evident that the ſhadows will be projected as falling towards the ſpectator, and that they will always be larger than the objects. I draw the ſhadows which are made by the ſun, having the ſame degree of elevation behind the ſpectator, which will make the ſhadows fall nearer the horizontal line, and leſs than the objects. The perpendicular from H, Fig. 106, muſt be let fall to L; and the angle HIL muſt be the ſame as HIK in the former caſe; and then, if the point L be made uſe of, inſtead of K, as theſe points are ſituated on the contrary ſides of the horizontal line, the ſhadows of objects will be repreſented as they would be made by the ſun placed on the back of the ſpectator, and this poſition of ſhadows is generally more agreeable in a picture than the other.

To find in this manner the ſhadow of the line ab in the ſolid X, Fig. 108., I draw Ha to the foot of it, and Lb to the top, interſecting each other in c, the place of the ſhadow. Finding in the ſame manner where the ſhadow of the line ef, behind the body, would fall, I join that point and c by a line which cuts the ſide of the figure in d, where only the ſhadow begins to be viſible. Thus, gaining e, c, and d, the ſhadow of the ſolid is completed.

In both theſe caſes, the ſun is ſuppoſed to be ſituated either before or behind the picture. If it be in the plane of the picture, it is evident that its place muſt be in the horizontal line, at an infinite diſtance from the point of ſight; conſequently all lines proceeding from it may be ſuppoſed parallel to one another. To draw the ſhadows of objects made by the ſun in this ſituation, draw lines parallel to the horizontal line from the foot of the object, and interſect them by lines making angles with the horizontal line equal to the ſun's altitude, as in Fig. 109 and 110.

CHAP. II.

Of the Application of Perſpective to Architecture, from Mr. Kirby.

Fig. 111.

I. *To put a* Tuscan Pedestal *into Perſpective.*

1. When one of its ſides is parallel to the picture, then the other ſide will be perpendicular to it; ſo that one rule will do in both caſes.

Let AB repreſent the bottom of the plinth in front. Now, from this one line AB, the appearance of the whole pedeſtal may be found: For, continue AB at pleaſure, and draw a line IK perpendicular thereto, and make IK equal to the height of the pedeſtal; then, upon IK, draw the capital and baſe in their proper proportions: This being done, continue lines from each moulding, which will form ſeveral rectangles, and thereby divide the planes 1234, 5678, into a ſort of network; then, by putting theſe planes into perſpective (as in the figure), we ſhall have ſufficient guides for drawing all the mouldings. But, to be more particular in the operation, make AD equal to AB, and cut off AE equal to AD, from whence the plinth may be completed. In like manner, for the die, draw the diagonals upon the top of the plinth, and, anywhere upon the edge hg, ſet off, the projection of the plinth as 1 9; then draw one line from 1 to C, and another line from 9 to L, which will give the projection 1 2: Then, if you draw a line through 2, parallel to hg, it will cut the diagonals ah, bg, and give the corners of the front ſide; and if you draw a line from b to C, it will cut the diagonal dc, and give the further corners; therefore, by drawing perpendiculars from a, b, c, we ſhall have all the edges of the die which can appear in this ſituation. As to the height of the die, or the height of the ſeveral mouldings, they may be found by drawing a perpendicular from the neareſt corner of the repreſentation, as BH, and transferring thereon the ſeveral heights from IK, as in the figure; then, by drawing lines from the ſeveral points upon BH (which meaſures the height of each moulding), to the vaniſhing point of the diagonal ah, we ſhall have thoſe heights transferred unto the edge am of the die: Thus, ai and km are the heights of the baſe and cornice; ſo that by finding the appearance of the planes 1234, 5678, and drawing the mouldings therein, and by drawing the triangular planes at the corners, we may finiſh the whole repreſentation with the utmoſt eaſe and expedition.

But before we begin to draw out any object in perſpective, we muſt firſt conſider, whether the whole, or only a part of it, is to appear; and muſt ſketch out the ſize we intend it ſhall be of, or at leaſt, give one line for its greateſt dimenſion. Thus, if the whole pedeſtal is to appear, then give AB, which is neareſt the eye, and call it the utmoſt length of the plinth; but if only the top is to appear, then give HO, and call it the utmoſt extent of the cornice; then, by cutting off O r equal to OM, that is, equal to OH, we ſhall have the depth O r of the cornice, &c., from whence, and with the aſſiſtance of the plane 5678 (which is found exactly in the ſame manner as the plane 1234), we may complete the whole that is wanted.

Now, in order to do all this, it is neceſſary that the artiſt ſhould be able to apply the preceding rules with the greateſt readineſs, particularly that which teaches how to cut off one line equal to another line given.

2. When both ſides are oblique with the picture. Fig. 111.

In this figure let A be the neareſt corner of the plinth, AC, AB the length of two ſides, AG, AF, and AK, the height of the whole pedeſtal properly divided (that is, like BH in the laſt figure). Cut off AF, AG, equal to AB, AC, and draw the plinth and the diagonals upon the upper ſquare; then draw 1b parallel to AB, and make it equal to the projection of the plinth; then cut off 1c equal to 1b, and from c draw lines to H, and from 1 draw a line to C (the vaniſhing point in the diagonal 1 2), which cutting each other in 2, will give the edge

2 7; and by drawing lines from 2 to their vaniſhing points L and H, they will cut the diagonal kg, and thereby give the other corners of the die, as in the figure.

For the height of the mouldings, draw lines from 4 and 5 to C, which will cut 2 7 in the points required; by which means the triangular planes aik, fbg, &c., may be found, and from thence the mouldings may be completed.

Here alſo, if we want only the upper part, we may begin at the point 8, making 8E, 8D each equal to the length of the cap, &c.; then by finding the plane 5678, as in the former caſe, we ſhall have ſufficient guides for completing the figure.

Here let us obſerve, that when the pedeſtal has one ſide parallel to the picture, then the plane 1234 (Fig. 112.), which is a guide for the moulding, may be begun anywhere upon the edge ab; but when it is oblique with the picture, then we muſt begin from the neareſt corner, as a; and by attending to the figures, we may conceive that, in the firſt caſe, the mouldings in the directing plane are like the ends of mouldings cut off ſquare; but in the latter caſe, they are like mouldings cut off at what is called the mitre joint. And from hence we may alſo obſerve, that all the difficulty in putting mouldings into perſpective, lies in finding the little planes 1234, &c.; and therefore the reader ſhould conſider them attentively before he proceeds any farther.

II. Of Circular Mouldings, &c.

The method for determining the appearance of circular mouldings is much the ſame as that for finding the repreſentation of ſtraight mouldings, viz. by imagining a plane to paſs through the mouldings in a perpendicular manner, and then putting that plane into perſpective, as in the two laſt figures.

Fig. 113.

1. *To put a* Tuscan Base *into Perſpective.*

Give one line for the width of the plinth, and draw out the proper projection of the mouldings and the plane ABCD; then cut off the oblique ſide equal to the front, and complete the plinth; after which draw the diameters and the diagonals upon the top of the plinth, as in the figure, and then draw the repreſentation of a circle for the ſeat of the lower torus. Again, for the bottom of the ſhaft of the column, draw the perpendicular HL; and from a, where the diameter ae cuts the plinth, draw ad parallel to H; then make ad equal to the height of the mouldings AD, and from d draw a line to C, which will cut HL in I; then will I be the centre of the ſquare for the bottom of the column: Therefore, upon the upper edge of the plinth, and from the point a, make a1 equal to the whole projection of the mouldings; then cut off ab equal to a1, and draw bc parallel to ad, which will give the little plane for the mouldings, within which draw the mouldings, and then we ſhall perceive that c is the middle of the neareſt edge of the upper ſquare: Therefore, through c draw a line EF, parallel to the edge B1 of the plinth; then from H draw a line through the centre 1, cutting EF in E, and then is cE half the width of the ſquare: Therefore make cF equal to cE, and from thence complete the ſquare, and within it draw the repreſentation of the circle, as in the figure. Finally, from the extremity of each circle draw the two oblique lines 2, 3, which, together with the little plane for the mouldings, &c. will be ſufficient guides for completing the whole baſe, as was propoſed, which is evident by inſpecting Figures 114. and 115.

As for making columns, &c, of any given proportion, or at any diſtance, the rule for cutting off a line in any given proportion, is ſufficient for that purpoſe.

Fig. 116.

2. *To put a* TUSCAN CAPITAL *into Perſpective.*

Let K be the centre of the ſquare for the bottom of the capital. Through K draw a line AB, parallel to the horizontal line at pleaſure, and from f and e of the baſe draw lines parallel to the axis HI of the column, cutting the above line in E and F; then is EF the diameter of the column: therefore diminiſh it in its proper proportion, as 3 4; then is 3 4 the diameter of the neck of the column. Therefore with the line 3 4 draw the appearance of a ſquare, and in that ſquare draw the repreſentation of a circle, as before directed; ſo ſhall we have a guide for the under part of the capital. Again, make KI equal to KF (that is, equal to half the diameter of the column), and through I draw GH parallel to AB; then make IG, IH, equal to KA or K B (that is, equal to half the diameter of the top of the abacus), and then with the line GH draw the appearance of another ſquare, which will repreſent the top of the abacus. Finally, from C draw a line through I, cutting the edge 5 6 in a, and from a ſet off a 1 for the projection of the capital, and draw the little plane abcd for the mouldings, as before; from whence the remaining part of the capital may be completed, as in Figures 117. and 118.

I ſhall juſt mention a method for finding the point where the diagonal of a ſquare will be cut by a circle inſcribed in that ſquare, which may be of uſe in this and in ſome other caſes. It is this: (Fig. 113.) Divide the edge BG into ſeven equal parts; then ſet one part from each corner, as B4, G5, and draw lines to C, which will cut the diagonals in the points required. I do not ſay this is mathematically exact; but I preſume it is near enough for the intended purpoſe.

Theſe are the moſt ſimple, as well as the moſt general methods I can think of, for mixed mouldings; and I believe any perſon who is but tolerably ſkilled in drawing, will find them ſufficient for his purpoſe upon all occaſions.

3. *To find the Repreſentation of a* CORINTHIAN CAPITAL.

Fig. 121.

Let AB be the diameter of the under part of the capital, and let ca be the centre or axis of the capital, properly divided, for the height of its leaves, volutes and abacus. From the line AB, which is given, find the appearance of a ſquare, in which draw the diameters and diagonals, and then the repreſentation of the circle, which will determine the places for the ſtalks of the great leaves, as repreſented by the dots: Again, through o draw bd, parallel to AB, and make ob, od, each equal to half the under part of the abacus; with which line bd draw the appearance of another ſquare, and divide it like the plan 1234, Fig. Z. of the under part of the abacus, and then draw the repreſentation of it as in the figure. Again, through a draw another line parallel to bd, and make it equal to the upper part of the abacus; then, by finding the repreſentation of the ſquare abcd, Fig. Z. we may draw the appearance of the upper part of the abacus, and from thence complete the abacus as in the 119th Figure. Finally, find the middle of each face of the abacus, as n, e, and draw lines, as n, 1, &c., to the correſponding points at the bottom of the capital; then find the height of the leaves, by drawing lines from C through the dots in ca, till they cut 1 n in 2 and 3; after which draw the baſket, then by a nice eye complete the capital, beginning as it is exemplified in the figure. The lines drawn from the corners of the upper and under ſquares will ſerve as guides, to prevent our giving the leaves too much projection. In Figure 120. the capital is completed, and the Figures X and Z are added, to explain the thing more fully; one of which is the plan, and the other half the profile, of a capital.

Here it is neceſſary to take notice, that upon account of bringing the diſtance of the picture within the compaſs of each plate, and to make the figures as large as poſſible, ſome of them have not that agreeable ſhape which could be wiſhed; but if the reader will chooſe a greater diſtance, and follow theſe rules, he will find every objection of this kind, that may ariſe, immediately vaniſh.

III. Of Columns Parallel and Oblique with the Eye.

Fig. 122.

1. Let it be required to find the appearance of two columns in front, and let ab be the diameter of the under part of the plinth, and c the centre of the column.

Continue ab at pleaſure; and anywhere upon it, as at A, draw a line AB perpendicular to Ab; upon which line ſet the ſeveral heights for the baſe, capital, entablature, &c. Then from c, the centre of the column, draw a line cd parallel to AB, and from the ſeveral diviſions upon AB draw lines parallel to the horizontal line, which will cut cd, and give the heights of the baſe, capital, and entablature. Now, having got the ſeveral heights, we are to conſider cd as the axis of the column (that is, a line which paſſes through the middle of it); and then, at every dot, make a ſquare, equal to the diameter of that part of the column, &c. which that dot ſtands for: Thus, ab is the under part of the plinth, and by means of H and the diagonal 1 2, we may complete the firſt ſquare. So alſo r is the ſquare for the bottom of the ſhaft, p for where the column begins to diminiſh, t for the top of the ſhaft, and o for the top of the abacus; and therefore, having got theſe ſeveral ſquares, we ſhall have ſufficient guides for completing a column of any order. Again, for the other parallel column ex: From c to e, and upon the line AB continued, ſet off the diſtance which one column is from the other, and draw ex; upon which ſet off the ſeveral heights as before, from the line AB, and then find the ſeveral ſquares as before directed.

2. For Oblique Columns.

Upon Ab continued, ſet off the diſtance which theſe columns are from the corner column, and draw a line from e to C; then cut off em, en, equal to ef, eg; and from the oblique ſides of the ſquare e, draw lines to C, from whence the other ſquares m and n may be completed as before. For their heights, draw mo and nq parallel to ex, and from x draw xC, which will give their ſeveral heights.

Now, if we would put an entablature over the column, then the height of the architrave, frieze and cornice, may be drawn from their reſpective diviſions upon the line AB; and the appearance of the mouldings peculiar to each part may be found by the rules already laid down for that purpoſe. Or, they may be found thus: Let agbf, in the Figure Z, repreſent the upper part of the capital, x, belonging to the corner column; and we want to find the corner of its entablature, that is, the corner of the lower faſcia or frieze, ſet off the projection of the capital from n to e, and draw a line to C, which will cut the diagonal ab, and give the corner of the faſcia; ſo that, by drawing the line l from the dot in the diagonal, we ſhall have the corner propoſed. In like manner, the projection of the cornice may be found: Continue 7o (which meaſures its height) at pleaſure, and make o h equal to the real projection of the cornice; then through H draw a line from C, and from H draw a line through o, and where it cuts Ch, continued in i, will be the corner of the cornice, &c.

From hence, then, it is evident, that any number of columns may be drawn with the greateſt eaſe and expedition, let their ſituations be what they will; and from hence, alſo, we may obſerve, that any part of a column may be immediately produced upon the picture in the very place in which it is intended, without drawing out the whole, or any other part than that which is really to appear.

Deſcription of the PENTAGRAPH, *an Inſtrument for Deſigning.*

The common pentagraph, Fig. 123. conſiſts of four rulers or bars, of metal or wood, two of them from 15 to 18 inches long, the two other half that length. At the ends, and in the middle of the long rulers, as alſo at the ends of the ſhorter ones, are holes, upon the exact fixing of which the perfection of the inſtrument chiefly depends. Thoſe in the middle of the long rulers are to be at the ſame diſtance from thoſe at the end of the long ones, and thoſe of the ſhort ones; ſo that, when put together, they may always make a parallelogram.

The inſtrument is fitted together for uſe by ſeveral little pieces, particularly a little pillar, No. 1. having at one end a nut and ſcrew joining the two long rulers together, and at the other end a ſmall knot for the inſtrument to ſlide on. The piece, No. 2. is a rivet with a ſcrew and nut, by which each ſhort ruler is faſtened to the middle of each long one. The piece, No. 3. is a pillar, one end of which being hollowed into a ſcrew, has a nut fitted to it, and at the other end is a worm to ſcrew into the table; when the inſtrument is to be uſed, it joins the ends of the two ſhort rulers. The piece, No. 4. is a pen or pencil, or portcrayon, ſcrewed into a little pillar. Laſtly, the piece, No. 5.

is a braſs point, moderately blunt, ſcrewed likewiſe into a little pillar.

Uſe of the Pentagraph.

1. To copy a deſign in the ſame ſize or ſcale as the original, ſcrew the worm No. 3. into the table, lay a paper under the pencil No. 4., and the deſign under the point No. 5. This done, conducting the point over the ſeveral lines and parts of the deſign, the pencil will draw or repeat the ſame on the paper.

2. When the deſign is to be reduced, *ex. gr.* to half the ſcale, the worm muſt be placed at the end of the long ruler No. 4., and the paper and pencil in the middle. In this ſituation, conduct the braſs point over the ſeveral lines of the deſign, as before; and the pencil at the ſame time will draw its copy in the proportion required; the pencil here only moving half the length that the point moves.

3. On the contrary, when the deſign is to be enlarged to a double ſize, the braſs point, with the deſign, muſt be placed in the middle, at No. 3., the pencil and paper at the end of the long ruler, and the worm at the other end.

4. To reduce or enlarge in other proportions, there are holes drilled at equal diſtances on each ruler, viz. all along the ſhort ones, and half way of the long ones, for placing the braſs point, pen-

cil, and worm in a right line in them, *i. e.* if the piece carrying the point be cut in the third hole, the other two pieces muſt be put each in its third hole, &c.

RULES FOR PAINTING IN WATER COLOURS.

The method of painting Landſcape in tranſparent water colours is a modern art, and has only been brought to a ſtate of perfection within theſe few years. The fundamental principle of all landſcape painting, however, is the ſame: To produce in a picture a good effect, ſuch a harmony is to be obſerved, that all the parts may appear to be ſeen through the ſame medium, from the ſame point of view, and to be enlightened by the ſame rays of light. For a diſcordance in the perſpective, or in the aſſemblage of colours, is like putting ſeveral pictures on the ſame canvas, which neceſſarily divides the attention, and conſequently puts an end to all intereſt.

The colours neceſſary to be uſed are,

1. A grey tint, known by the name of Smyth's Grey.
2. Pruſſian Blue.
3. Red Lake.
4. Light Red.
5. Burnt Terra de Sienna.
6. Raw Terra de Sienna.
7. Yellow Ochre.
8. Gamboge.
9. Gall-ſtone.
10. Bt. Umber.

11. Sap Green.
12. Pruſſian Green.
13. Indian Ink.

The grey tint is compoſed of Pruſſian blue, red lake and yellow ochre; and as red, blue and yellow are the three priſmatic colours, by a proper combination of which any other colour may be formed, they are therefore the greateſt oppoſites in quality; and when combined in equal proportions, as in this tint, they deſtroy each other, and produce nearly a black.

This therefore appears to be the moſt proper colour for producing what is called the general effect of a drawing; ſince, in a landſcape ſo drawn, if there be afterwards put upon the different objects repreſented in it the proper colours they have in nature, ſo that the grey may appear through them, the reſult will be, that the blending of thoſe together will neceſſarily produce an effect ſimilar to that produced by the colour of objects blended with that of the atmoſphere.

MORNING.

Morning views ſhould conſiſt of large maſſes, ſuch as ſteep rocks, high mountains and deep valleys; for all large projections are beſt ſeen, and the contraſt of light and ſhade from ſuch objects is fineſt when the riſing ſun throws his long rays over the ſurface of the earth.

After the outline of ſuch a ſcene is properly drawn, take a little raw terra de Sienna, of the lighteſt tint, and waſh over the whole drawing from the bottom, ſoftening off the colour with water towards the top. For the ſky, take Pruſſian blue; and beginning at the top, ſoften off the tint with water as it deſcends towards the horizon. The light of the clouds ſhould not be left entirely white. For their ſhades, and the darker part of the ſky, the grey tint may be uſed. After this, waſh over the whole drawing, except the bright lights upon the fore and middle grounds with a light ſhade of the grey tint. When the drawing is nearly dry, come over it again, except the bright and ſecondary lights, with the ſame tint uſed before. After your drawing is quite dry, make out the ſhades of the different objects with different tints of the grey, increaſing the ſtrength of the colour as they become nearer to the eye. For thoſe objects that are at the greateſt diſtance, ſuch as mountains and valleys, a little blue may be added to the grey. For fields, meadows, hills, and plantations, in the middle ground, add a little ochre to the grey. The ſhades of thoſe objects upon the fore ground require the grey to be ſtrengthened with umber or Indian ink; and in many caſes, eſpecially for the dark ſhades of rocks and the trunks of trees, a mixture of Indian ink and umber is preferable.

When the effect of the drawing is thus produced, colour the different objects with their proper colours; and as the colours of objects repreſented in a morning view are more determined and more vivid than either at noon or in the evening, the tints of the colours uſed require to be ſtronger than in midday or evening ſcenes. The yellow uſed for the diſtance ſhould always be ochre. Raw terra Sienna is preferable for fields and meadows in the middle ground, and gamboge or gall-ſtone for yellow objects on the fore ground. The colour of trees on diſtant ground may be made with a mixture of Pruſſian green and gamboge. For thoſe on the fore ground, different mixtures of ſap-green, burnt terra de Sienna, gamboge and burnt umber are neceſſary. Rocks may be coloured with different ſhades of umber and light red, or Pruſſian blue and lake, according to the particular colour they aſſume, or the diſtance they are removed from the eye. For earthy ſandy roads or banks, burnt umber and lake, or light red alone, is a good colour. The graſs in the middle ground may be produced by touches of Pruſſian green. Graſs and moſs on the fore ground are made out with different touches of ſap-green, umber, burnt terra de Sienna, and gamboge, by the point of a large pencil having its hair ſeparated at the point, which is beſt done by a ſmall comb; and pencils uſed for this purpoſe ſhould be uſed for no other; for one ſtroke of ſuch a pencil, has the effect of many ſtrokes made with the point of a ſmall one. The light which ſhines through the blade of graſs, may be produced by carrying the colour of the graſs a little beyond the edge of the dead colouring.

NOON-DAY.

Confined ſcenes, elegant buildings, or ſcenes interſperſed with ſmall objects near the eye, are beſt ſuited for repreſentation when the ſun is at its greateſt height above the horizon. Clear and clouded ſkies have the beſt effect in ſuch ſcenes, or a ſingle light cloud floating in the air. The greateſt light of the cloud ſhould never extend quite to the edge, which ought to blend with the colour of the ſky. The colours of the objects ſhould be fainter, but of a warmer tinge than in a morning view, as the atmoſphere is commonly then filled with more vapours than in the morning, and conſequently the objects, although they have a warmer glow, have not the ſame diſtinct appearance. A little light red may alſo be mixed with the terra de Sienna uſed for the ſky.

Evening.

Large extenſive proſpects, interſperſed with maſſes of rocks and trees, in the middle ground, thrown together by the hand of Nature with wildneſs and confuſion, are beſt ſuited for ſcenes of this kind: For then the imagination, not being diſtracted either with the regularity of the works of art, or a multiplicity of ſmall objects, can dwell upon and contemplate with much more pleaſure that exquiſite variety of ſoft and pleaſing colours which then embelliſhes the clouds and the diſtant country.

After the outline is drawn, take a mixture of burnt terra de Sienna and a little gamboge, and waſh over the whole drawing with this tint, ſoftening the colour from the bottom towards the top: and, after the general effect of your drawing is produced with the grey, repeat the ſame tint in the ſame manner as before, but ſtronger. When the different objects are tinted with their different colours, and the drawing is nearly finiſhed, waſh the whole over with another tint of burnt terra de Sienna, which will cauſe that warm glow and tender colouring ſo predominant in evening ſcenes. All hard ſharp touches, and minute work in the fore ground, which in other caſes may pleaſe, ſhould in warm evening ſcenes be carefully avoided.

WATER.

The different ſorts of water which are repreſented in Landſcape are, 1ſt, The Ocean; 2dly, Lakes, or Dead Waters; 3dly, Rivers; 4thly, Rapid Streams; 5thly, Gentle Falls; 6thly, Caſcades. When the ſurface of the water is ſmooth, as in a calm upon the ſea, or in a lake, where there are no ſhades or reflections either from terreſtrial objects or from floating clouds, it is darkeſt at the fartheſt extremity, and grows gradually lighter as it approaches towards the eye. But if any object throw a ſhadow over the water, the neareſt part is then the darkeſt; and it grows gradually lighter as it retires from the eye. All the ſhades made in water ſhould alſo be perfectly horizontal when it is intended to repreſent a ſmooth ſurface. The dead colouring of all ſorts of water ſhould be in the grey tint. The colouring of it depends entirely on the colour of the ſky, or thoſe objects which are reflected from its ſurface.

The ſpray or miſt which riſes from the bottom of falls may be repreſented by waſhing with a clean wet bruſh all the ſhades which come to the ſide of the water. The direction of the ſtrokes or ſhades which mark the courſe of a current ſhould alſo change at every obſtruction. Faint tints of light red, in particular places, give the appearance of the bottom.

THE END.

MVSEVM BRITANNICVM

Fig. 1.

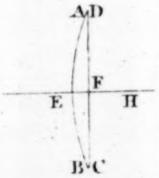


Fig. 2.

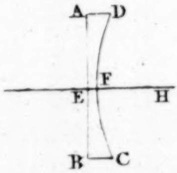


Fig. 3.

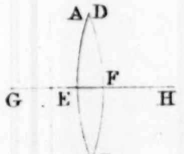
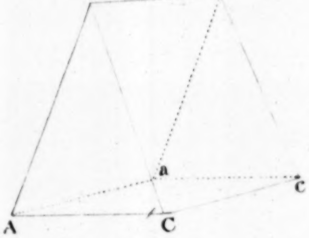
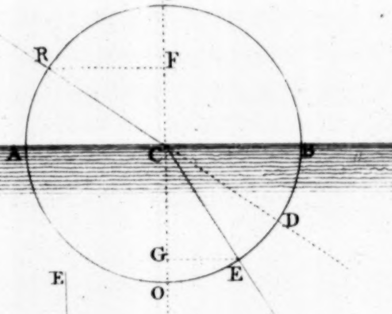


Fig. 7.



Fi

Fig. 10.



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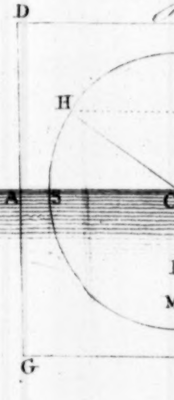

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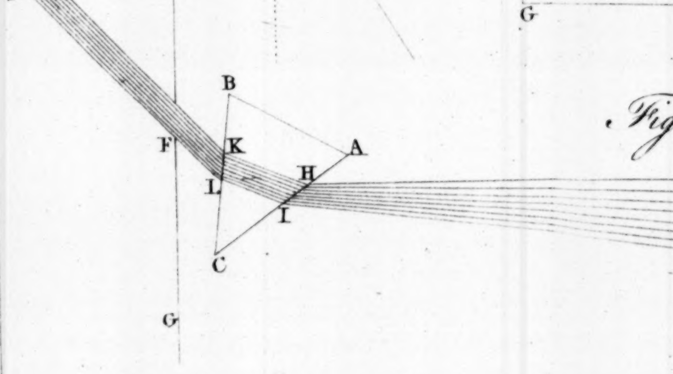

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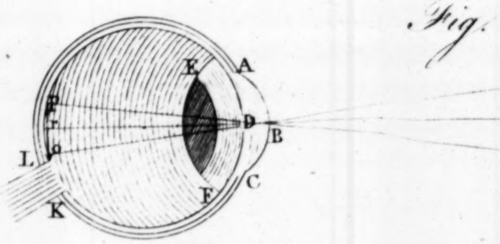
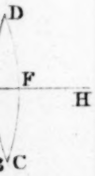


Plate I.

Fig: 3.



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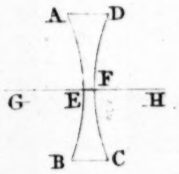

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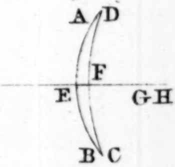

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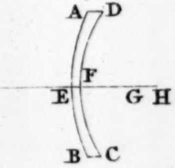

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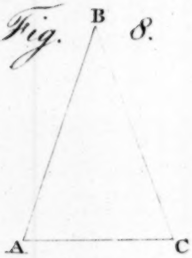

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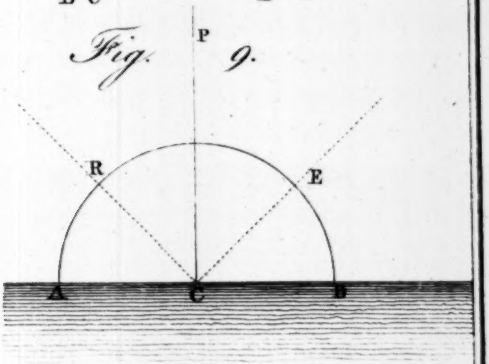

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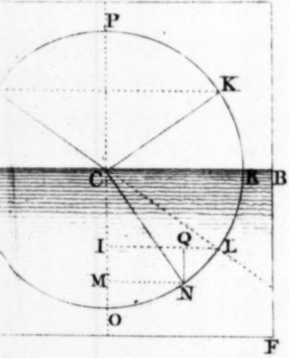

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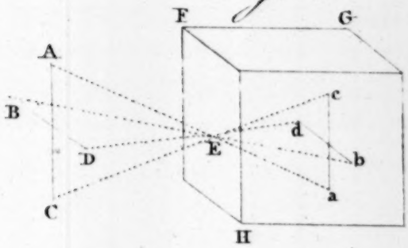

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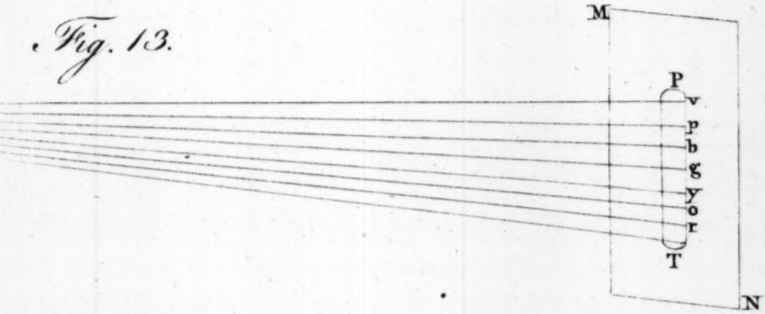

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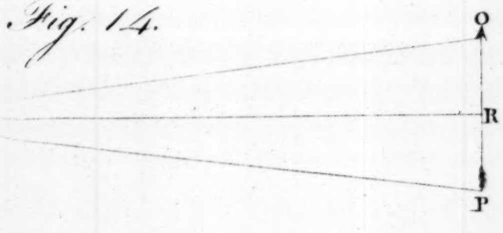

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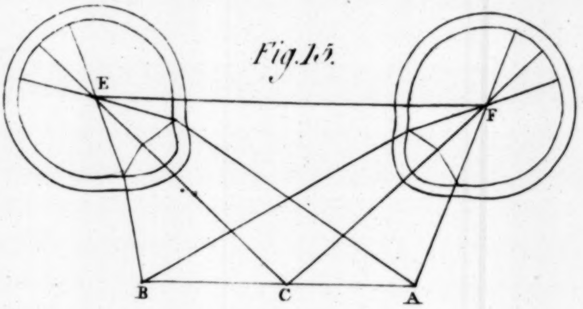

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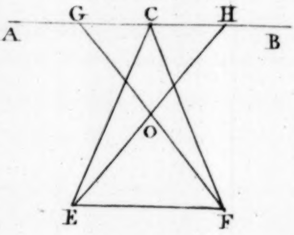

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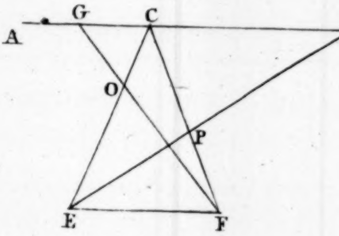

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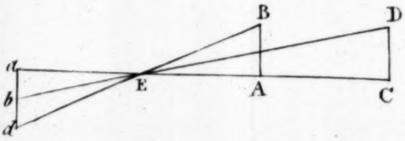

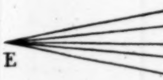

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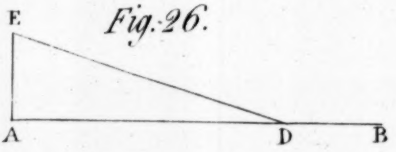

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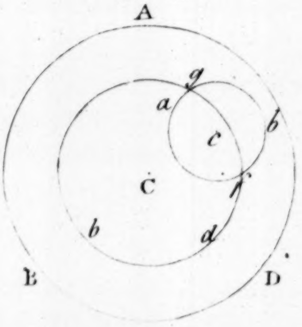

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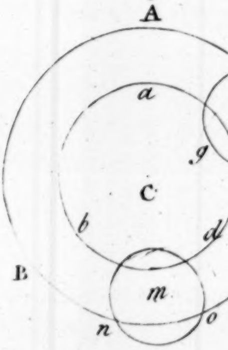

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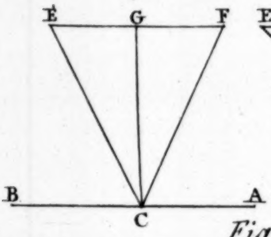

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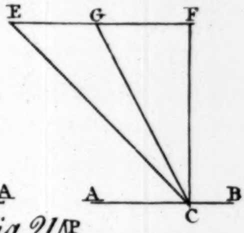

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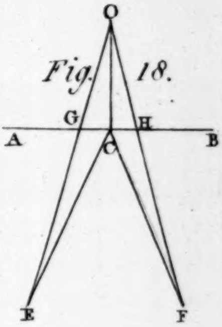

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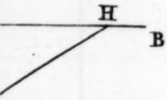

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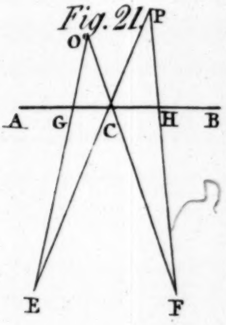

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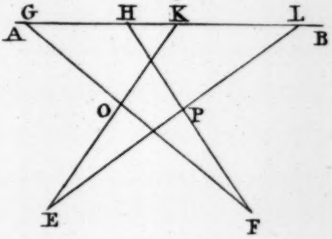

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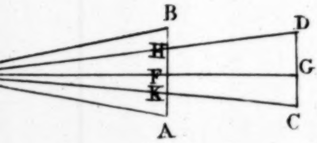

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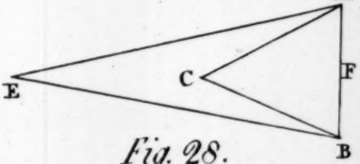

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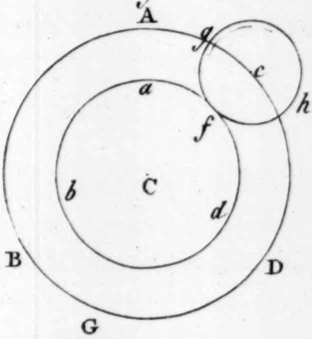

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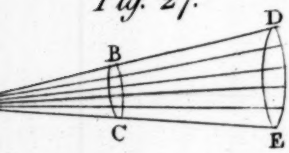

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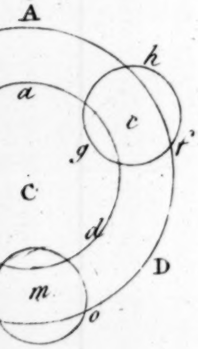

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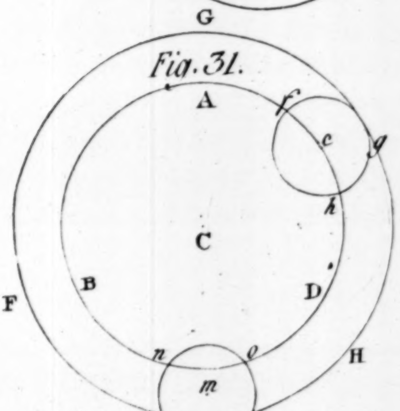

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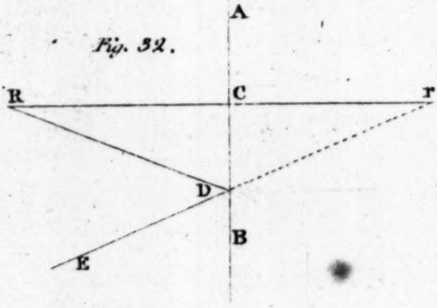

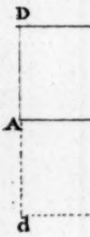

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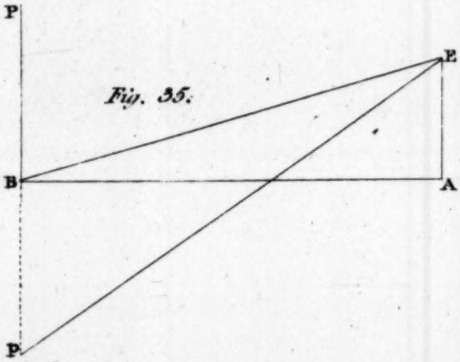

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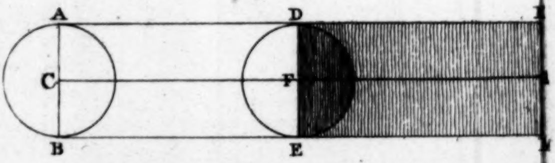

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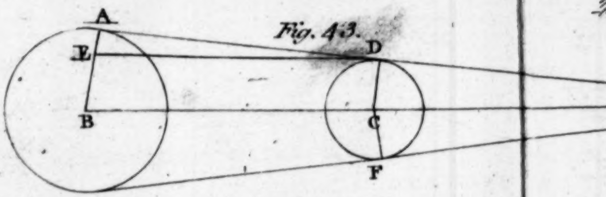

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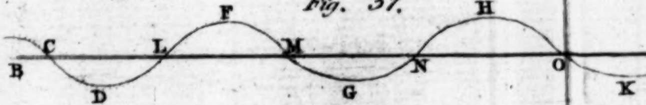


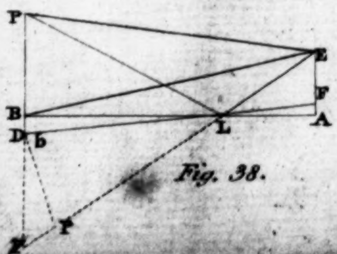
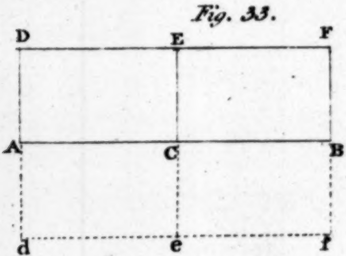


Fig. 38.

Plate III.

Fig. 33.



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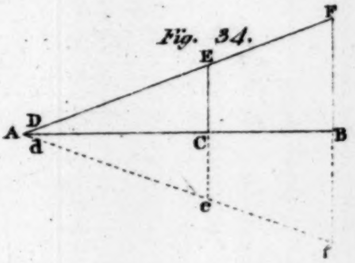

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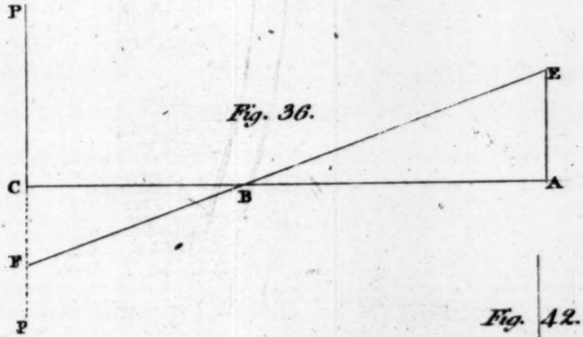

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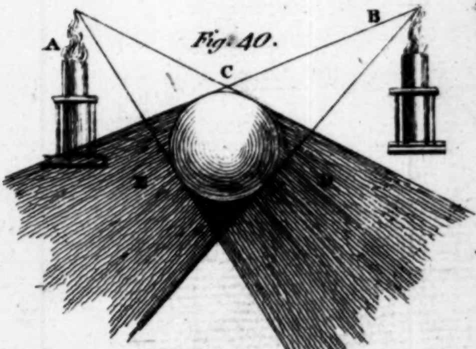

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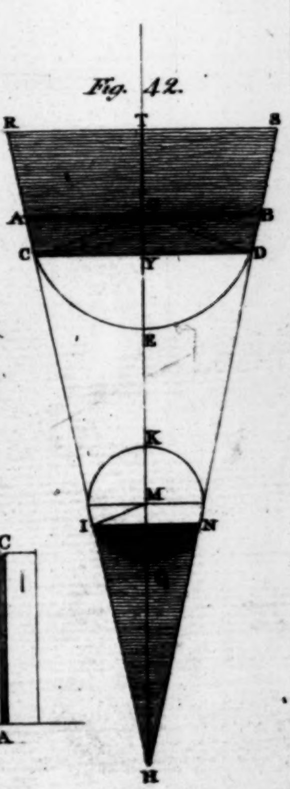

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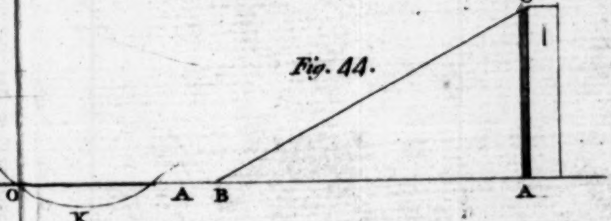

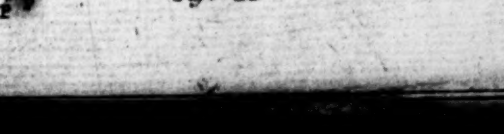

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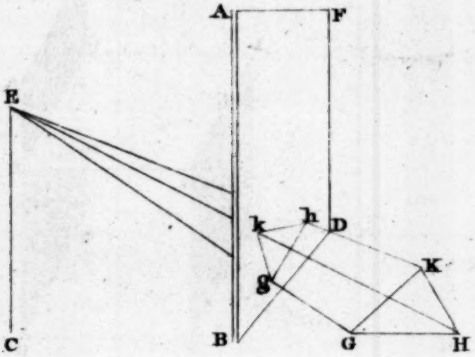

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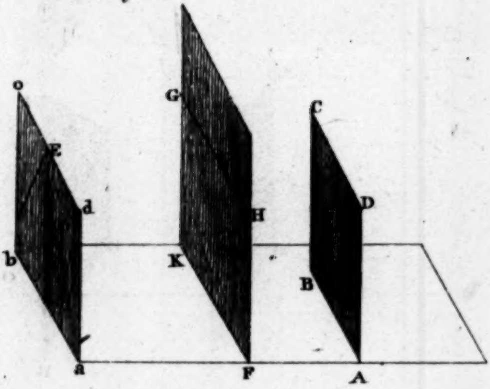

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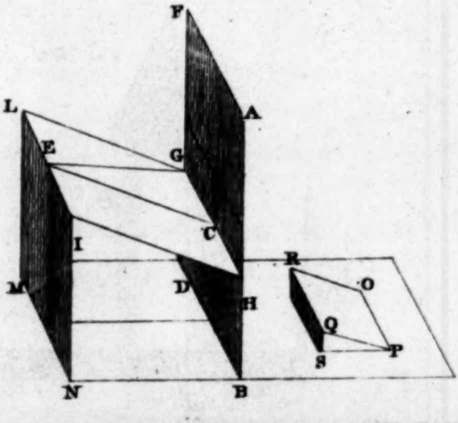

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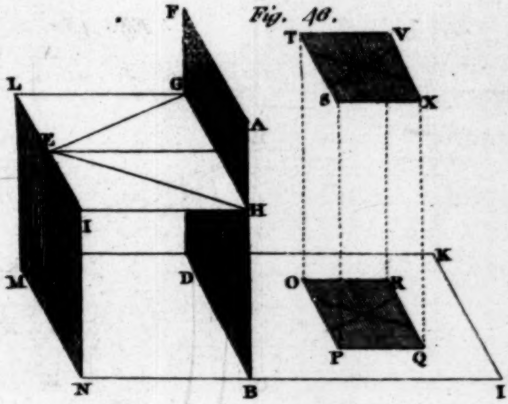

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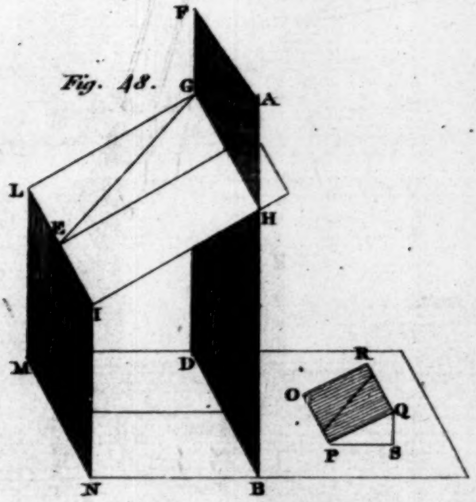

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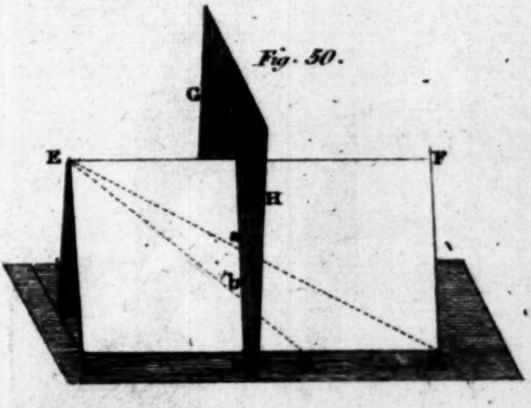

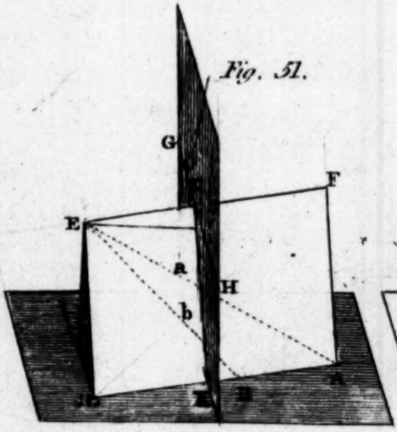

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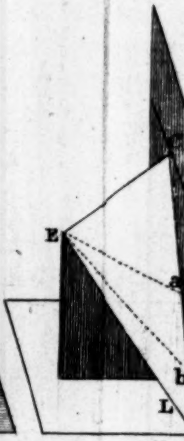

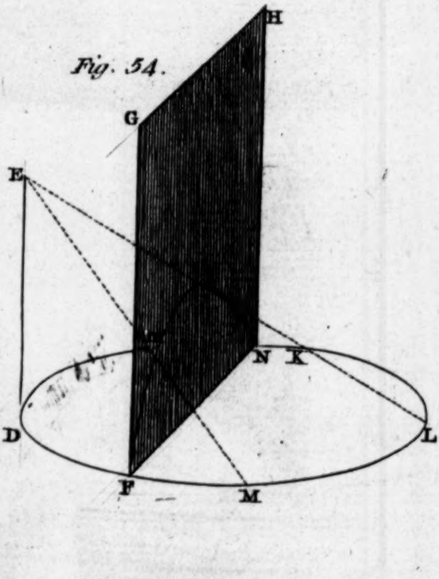

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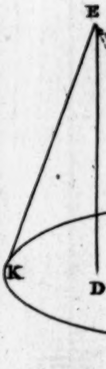


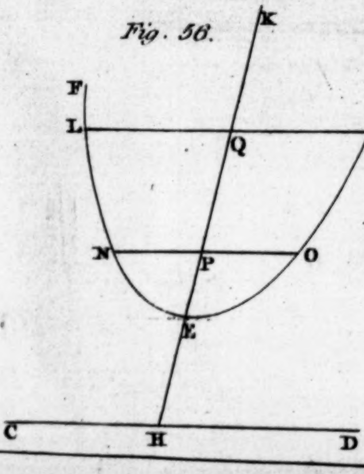
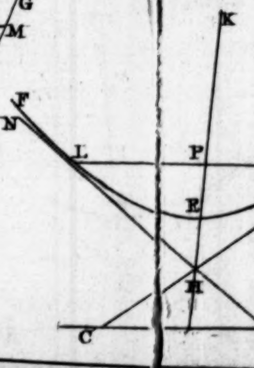


Fig. 56.

Fig. 57.



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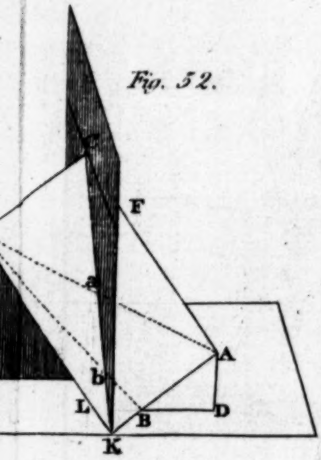
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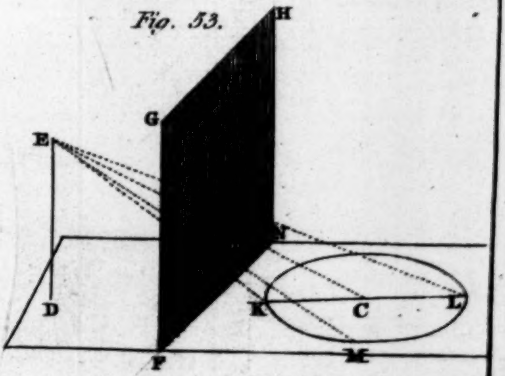
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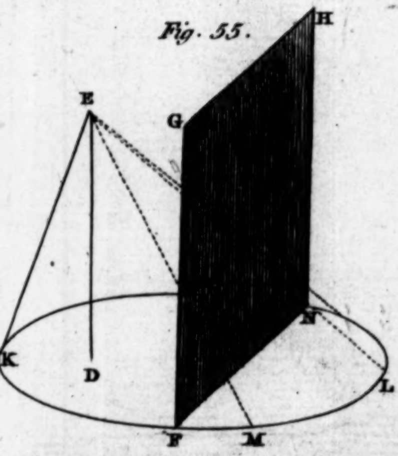
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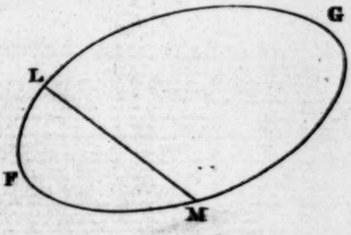
Fig. 58.



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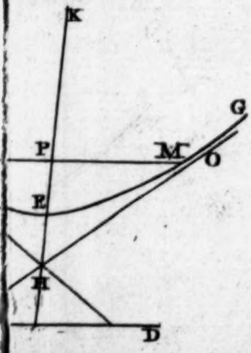


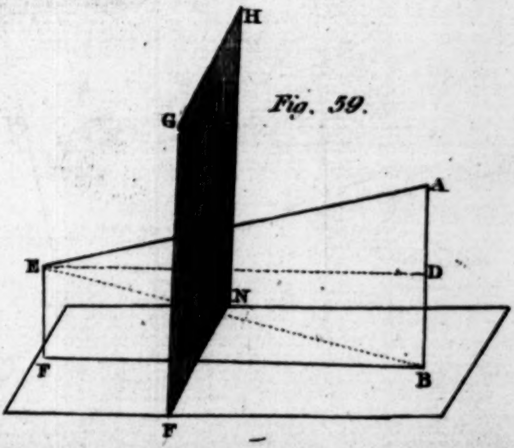
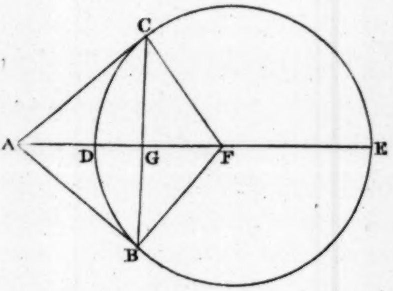


Fig. 59.

Fig. 60.



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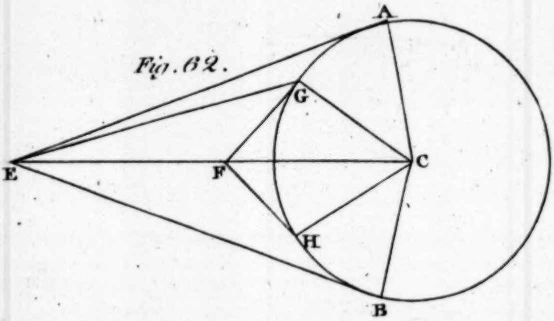

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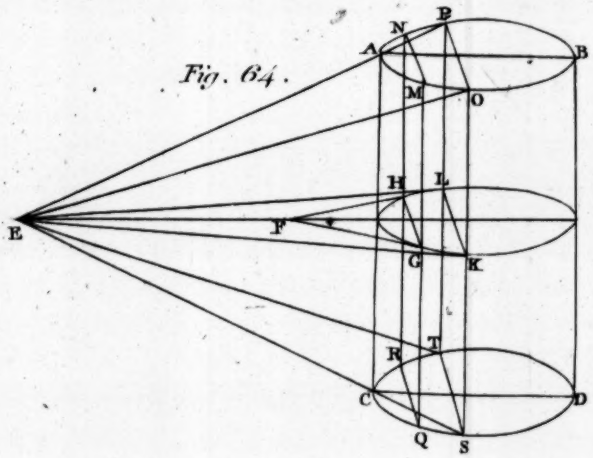

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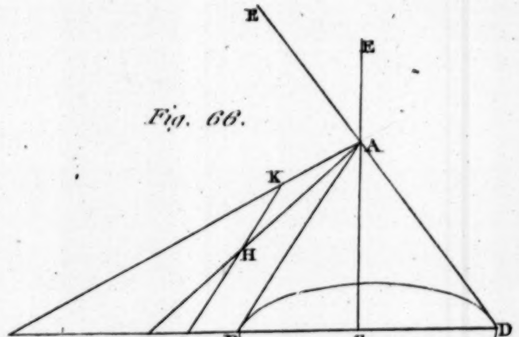
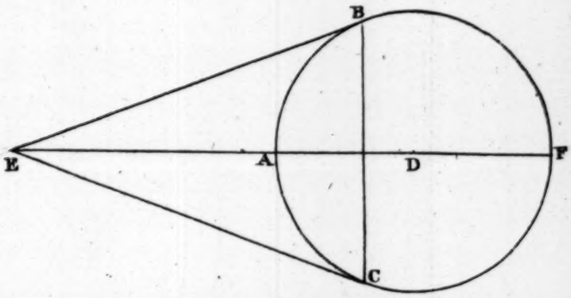


Plate VI.

Fig. 61.



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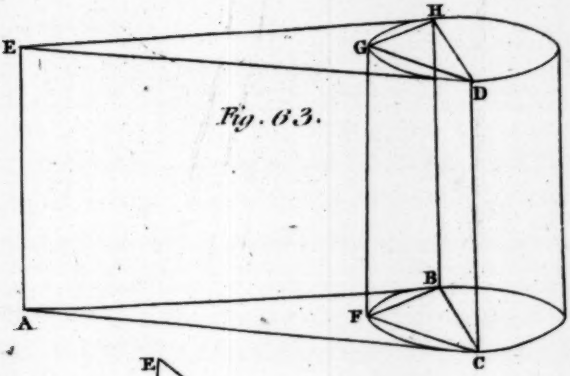

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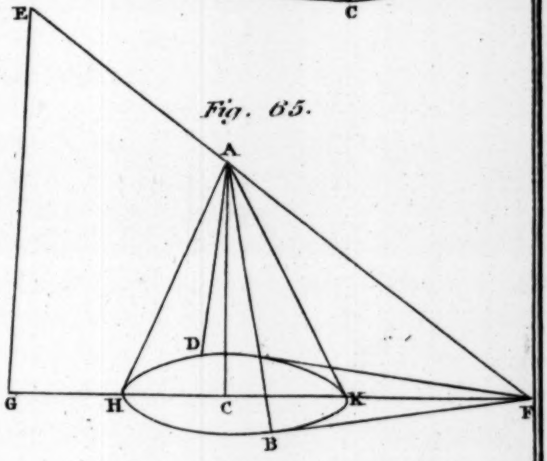

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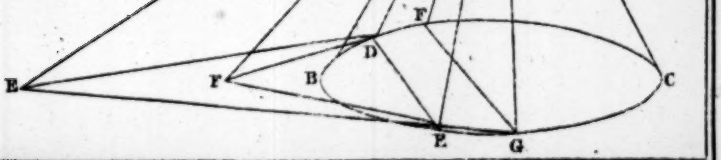

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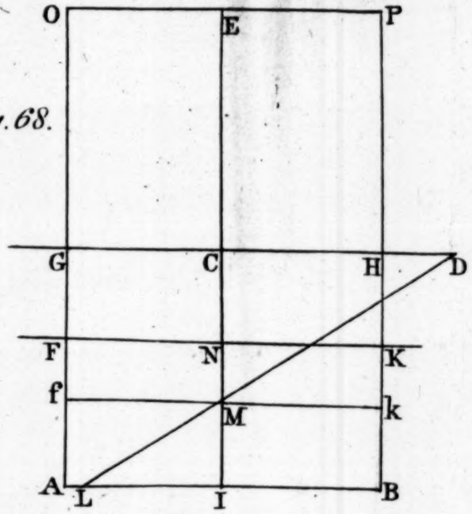

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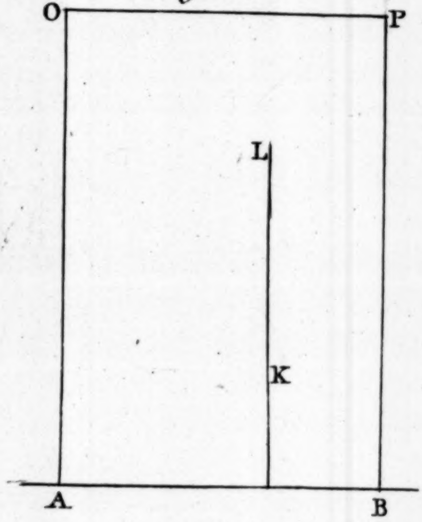

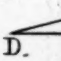

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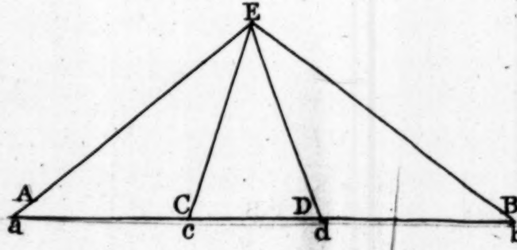

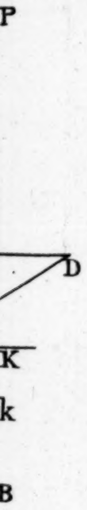

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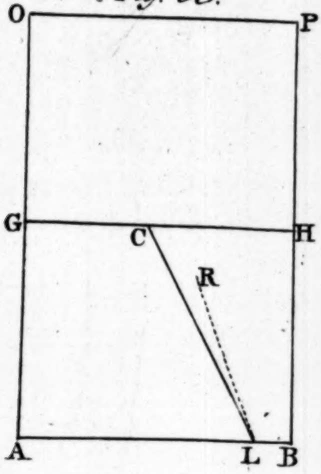

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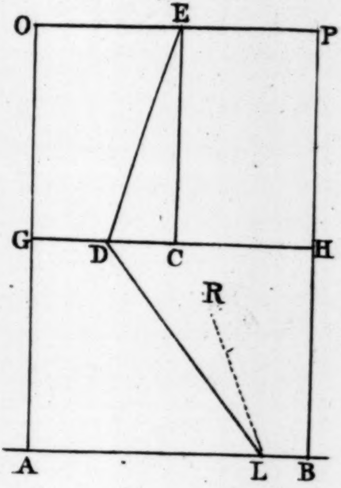

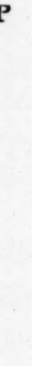

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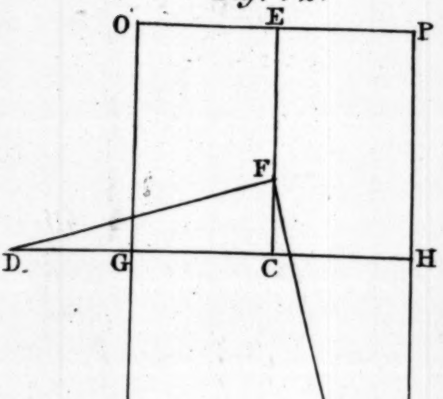

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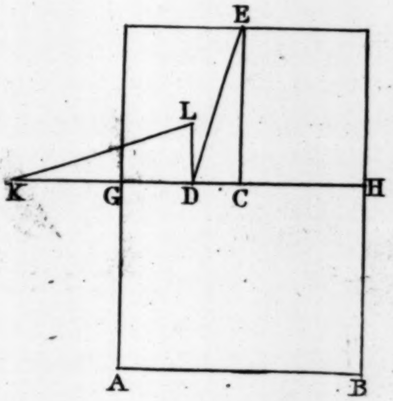

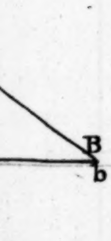

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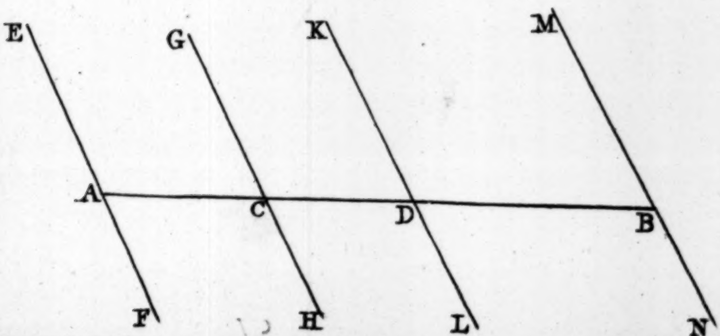

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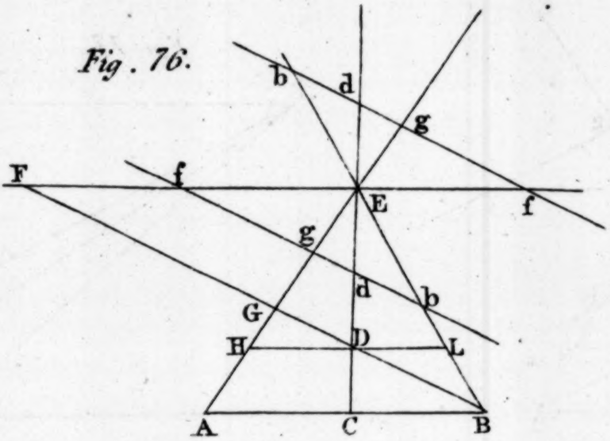

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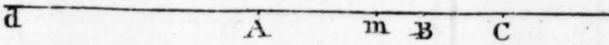

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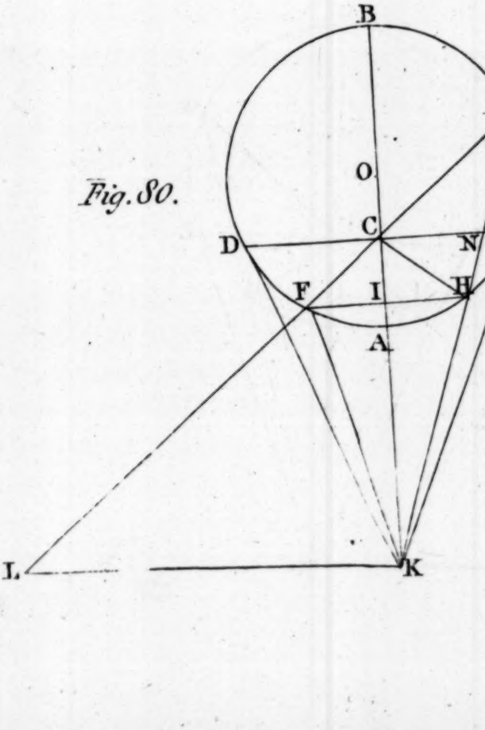

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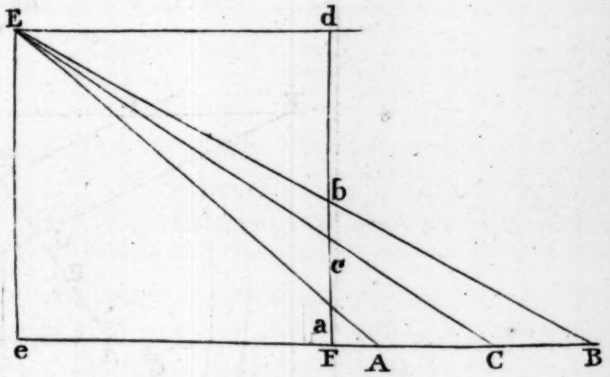

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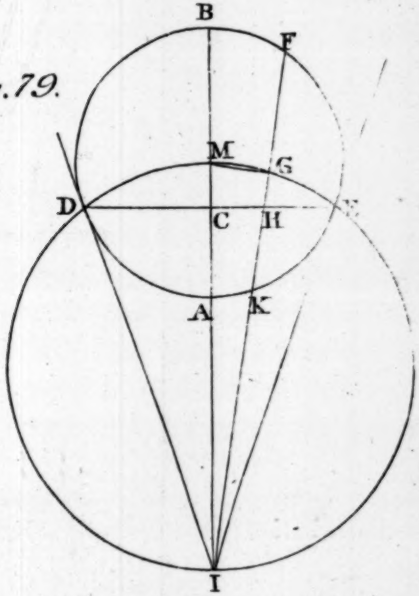

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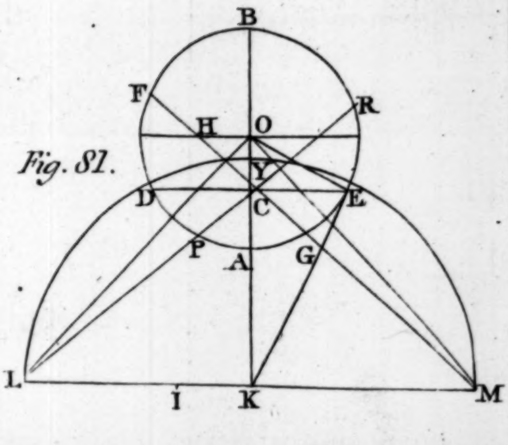

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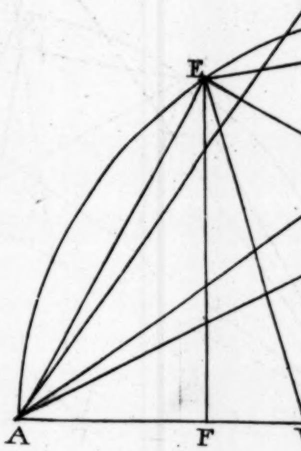

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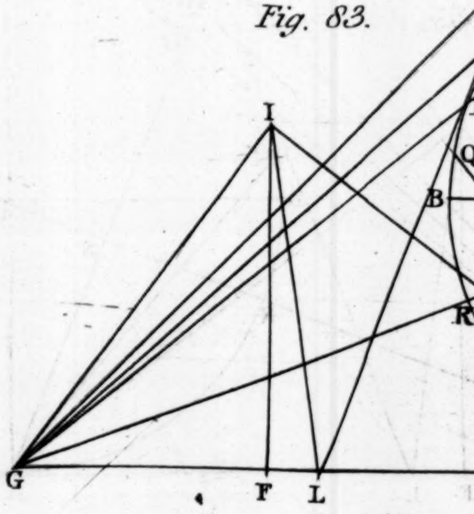
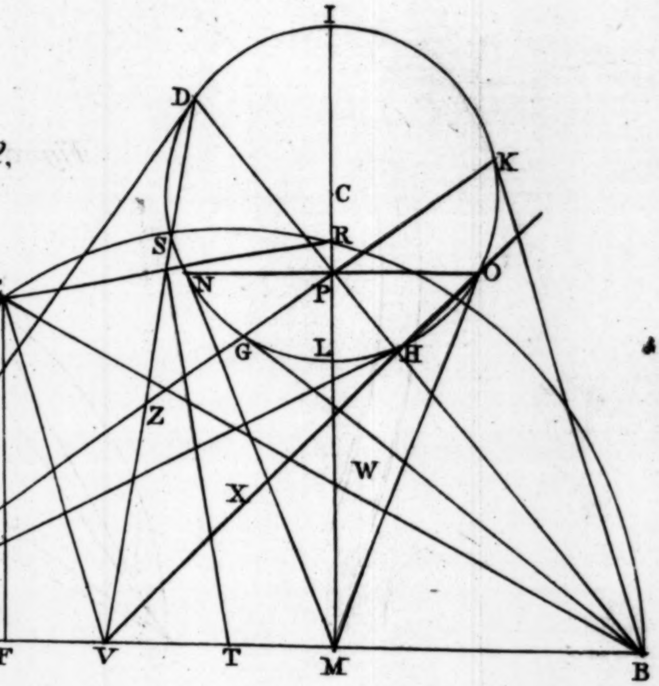
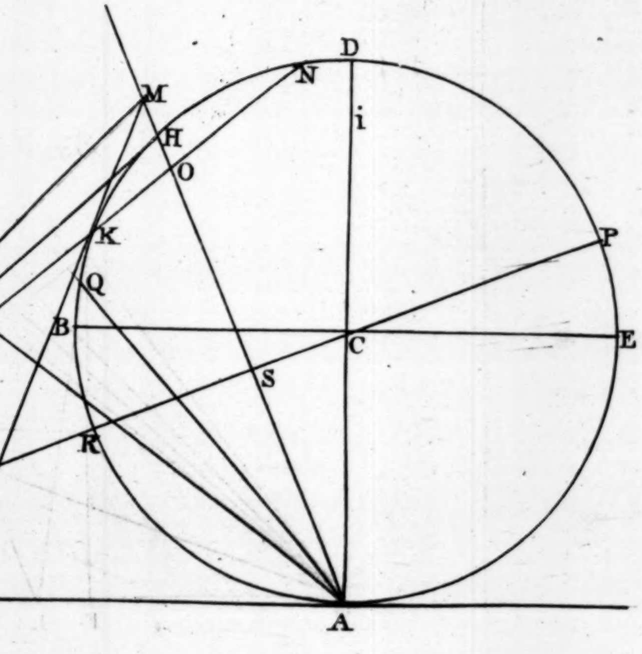


Plate IX





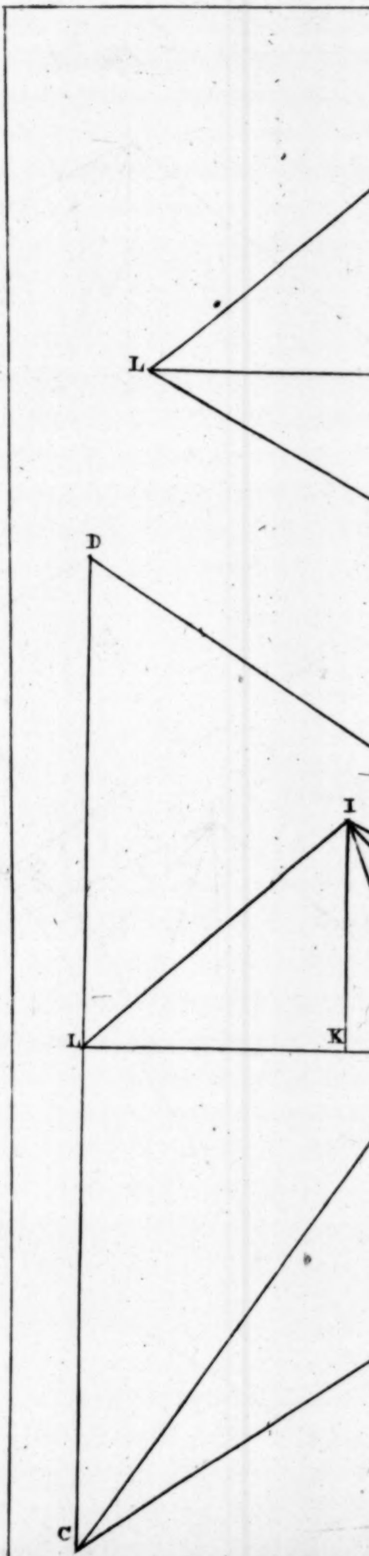

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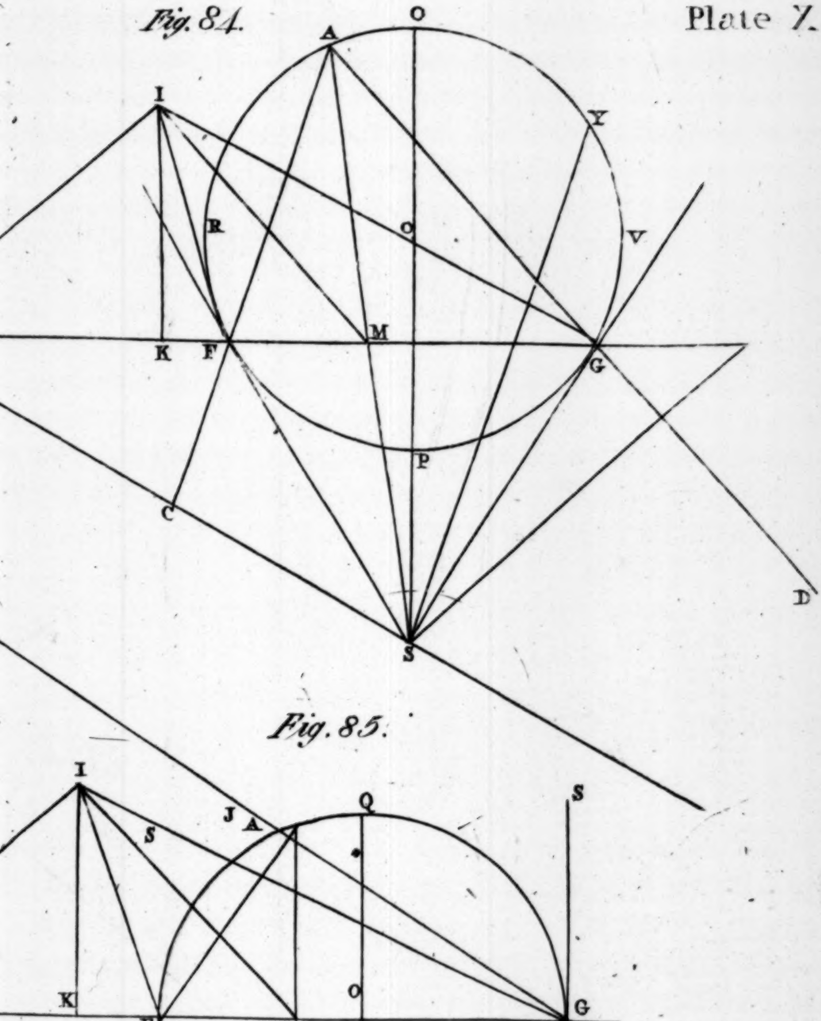
Plate Z.

Fig. 84.



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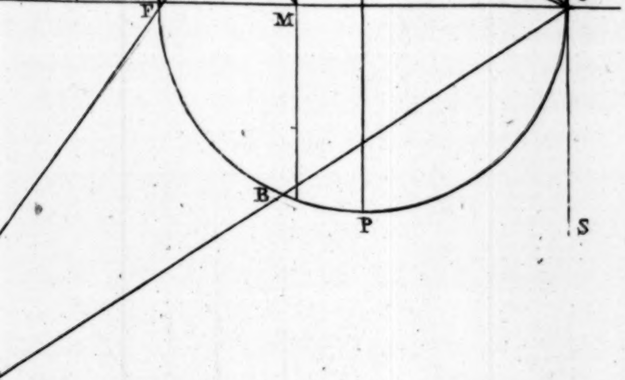

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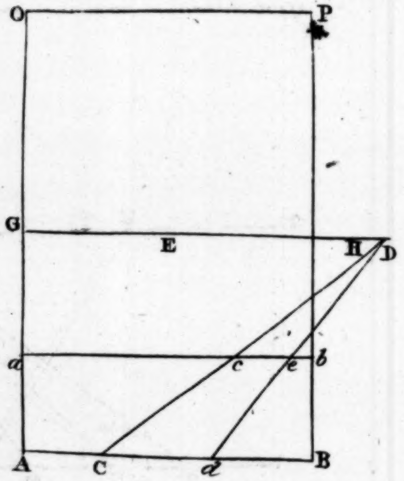

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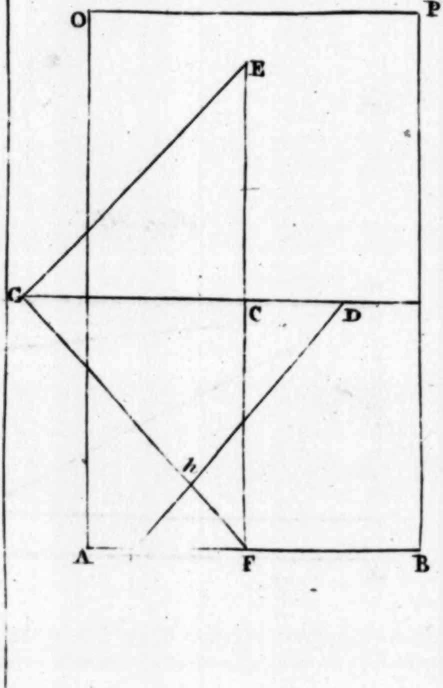
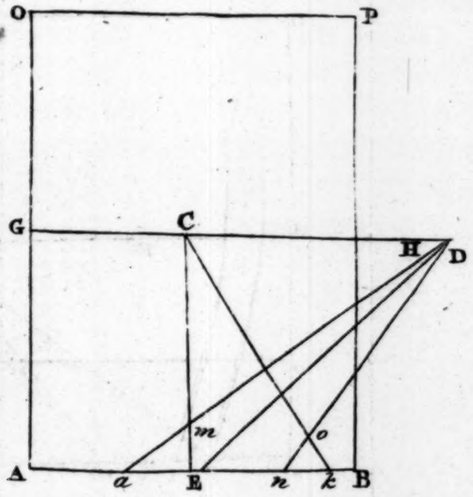


Plate XI.

Fig. 87.



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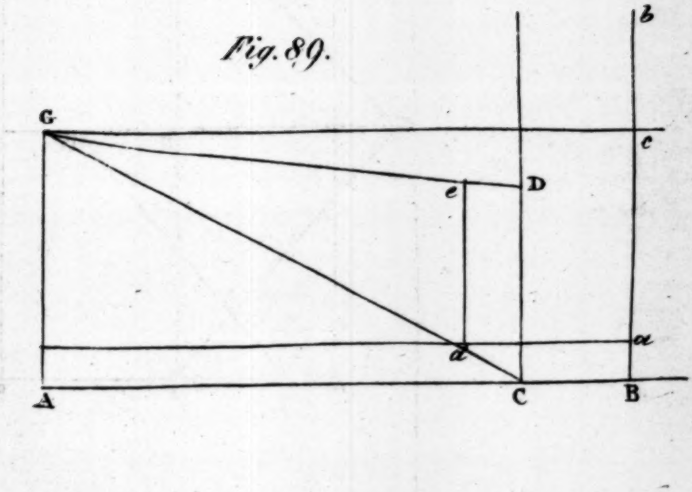

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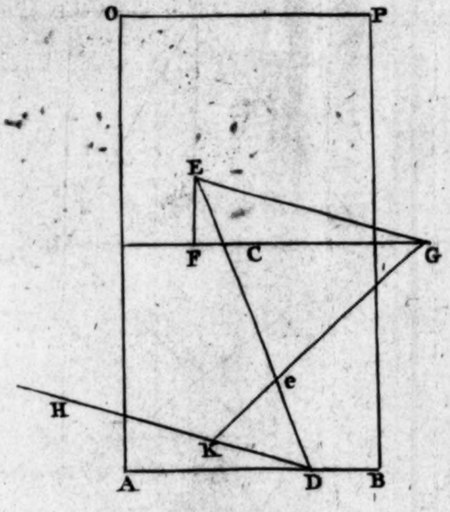

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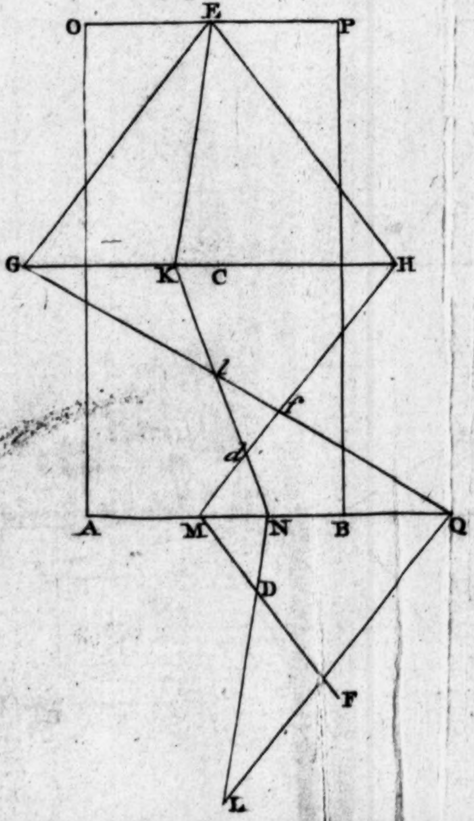
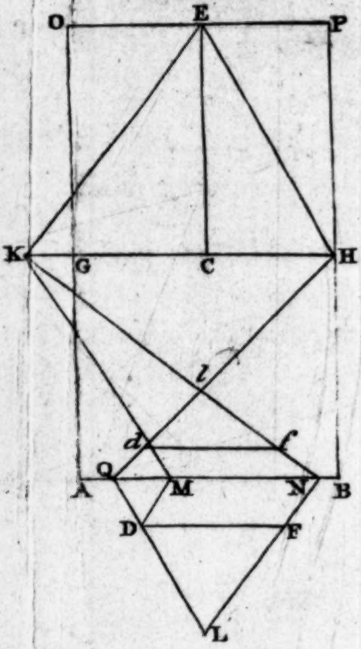


Plate XII

Fig. 91.



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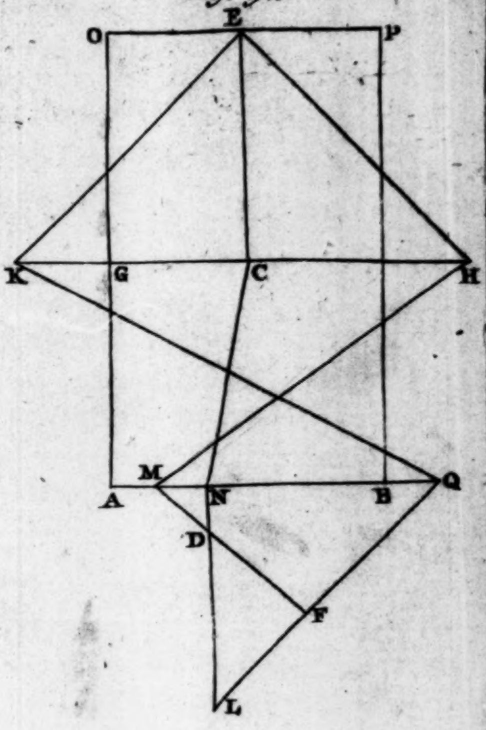

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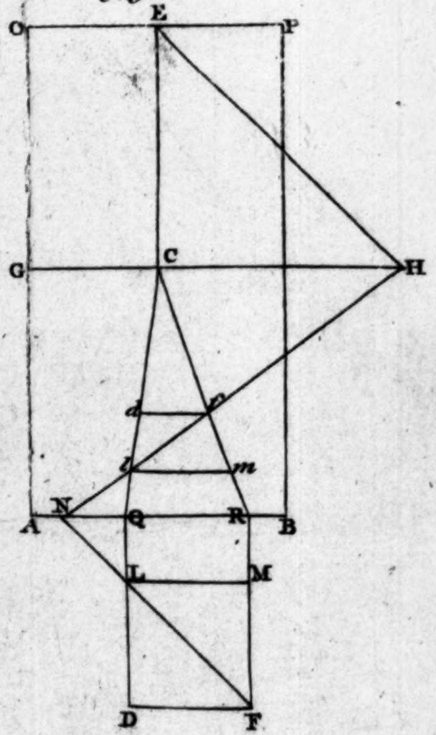

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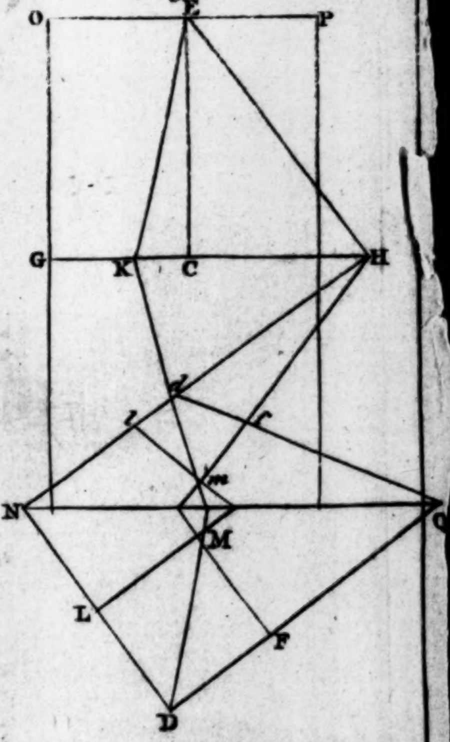

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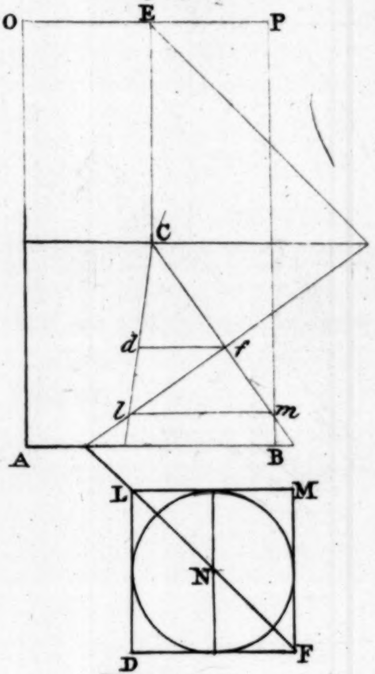

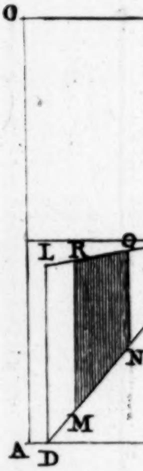

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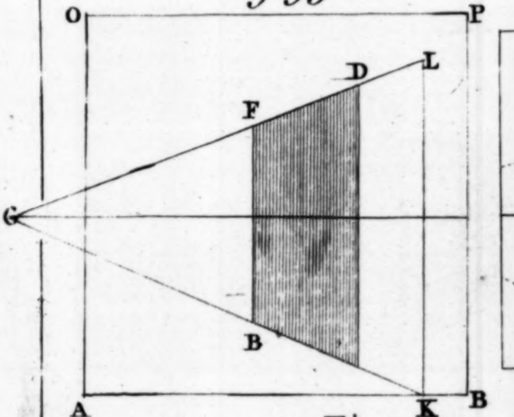


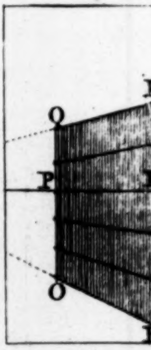


Fig.102.

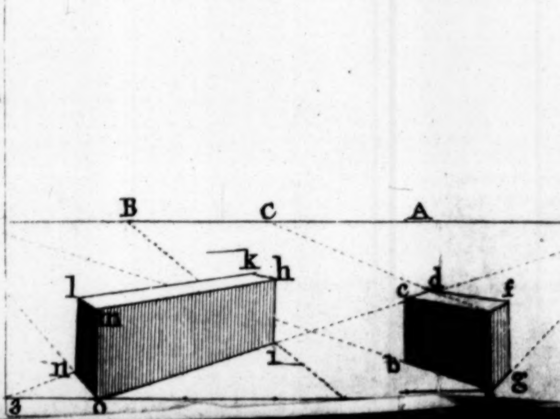


Plate 13

Fig. 97.

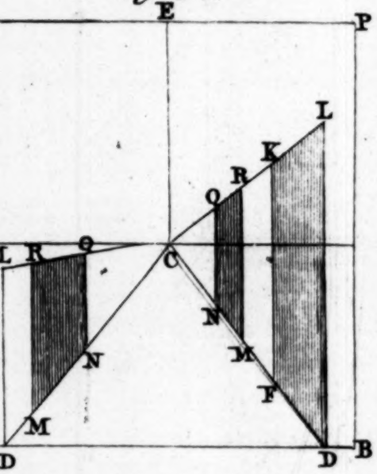
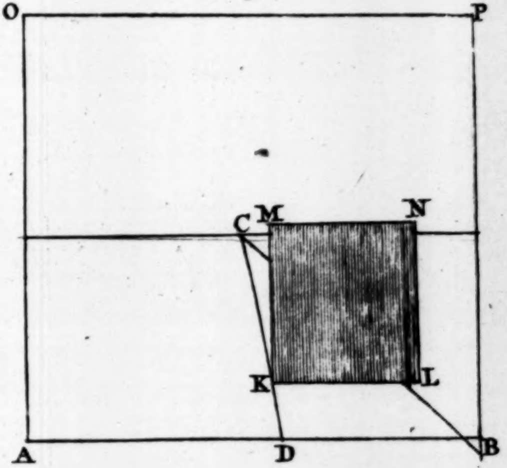


Fig. 98.



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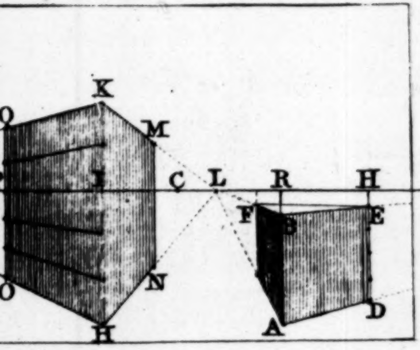

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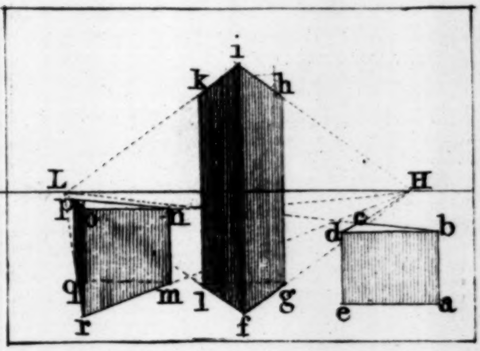

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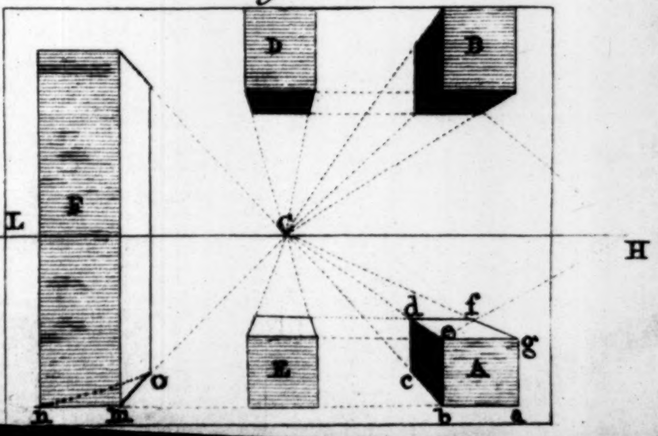

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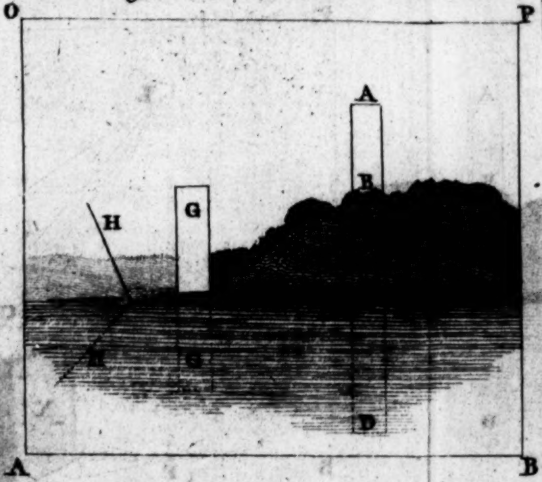

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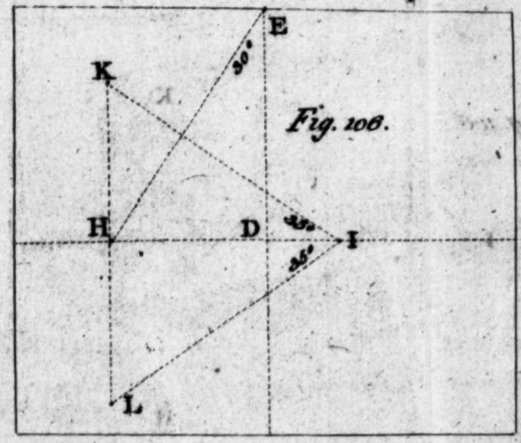

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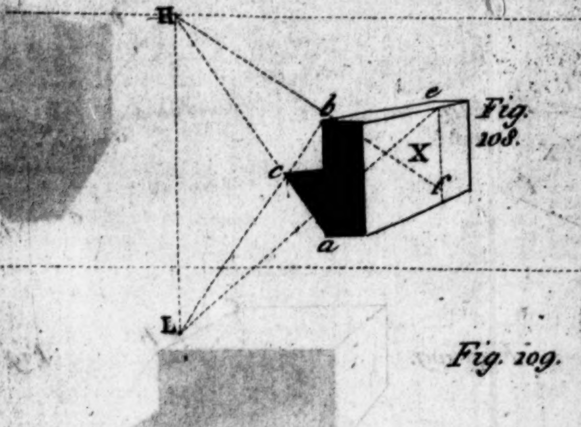

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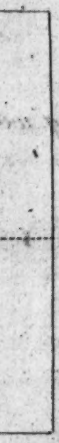

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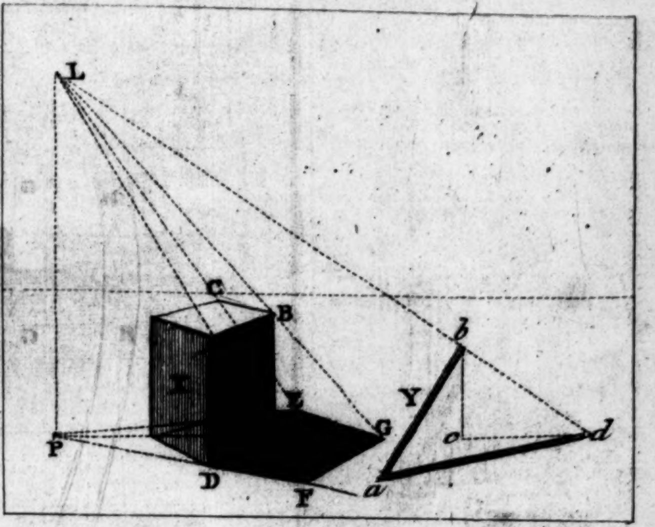

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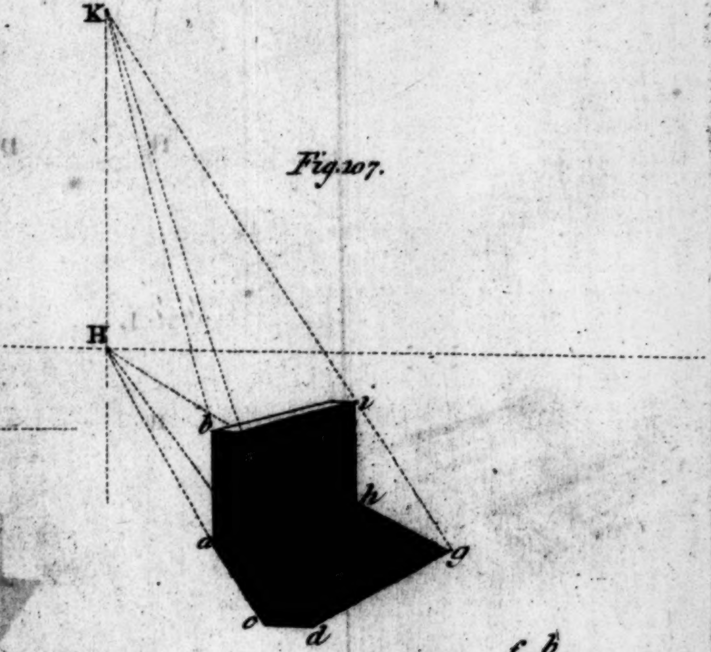

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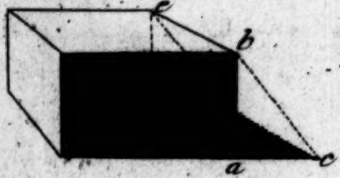

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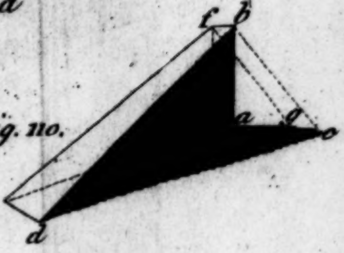

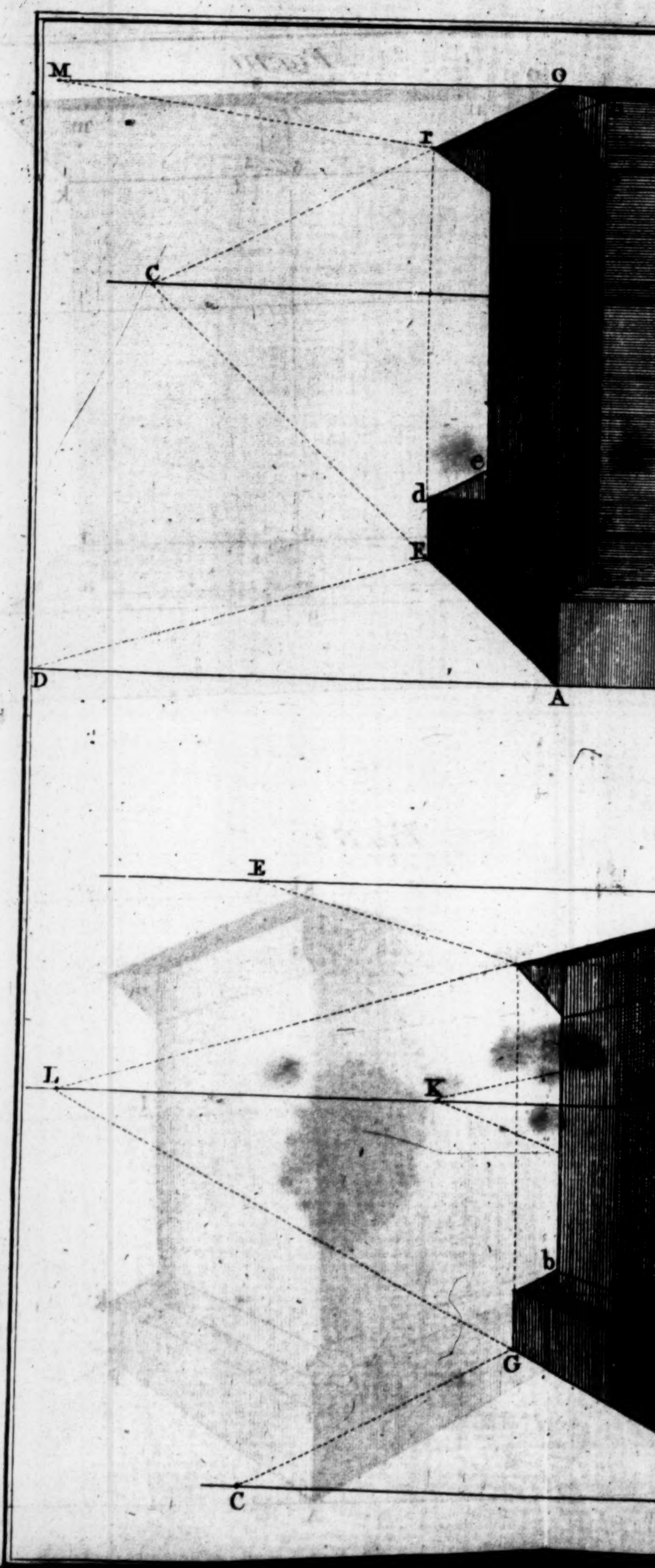
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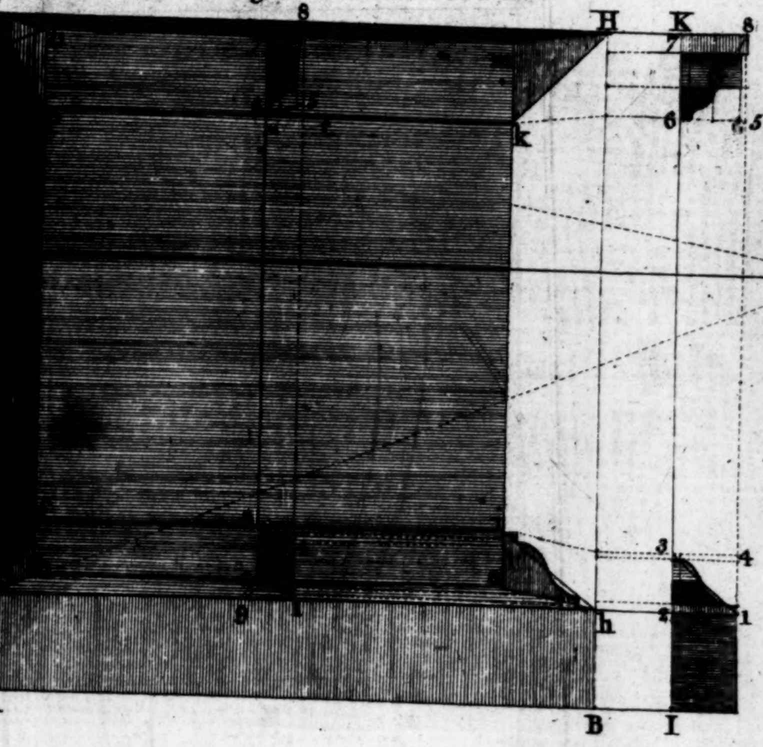
Plate. 15.

Fig. 111.



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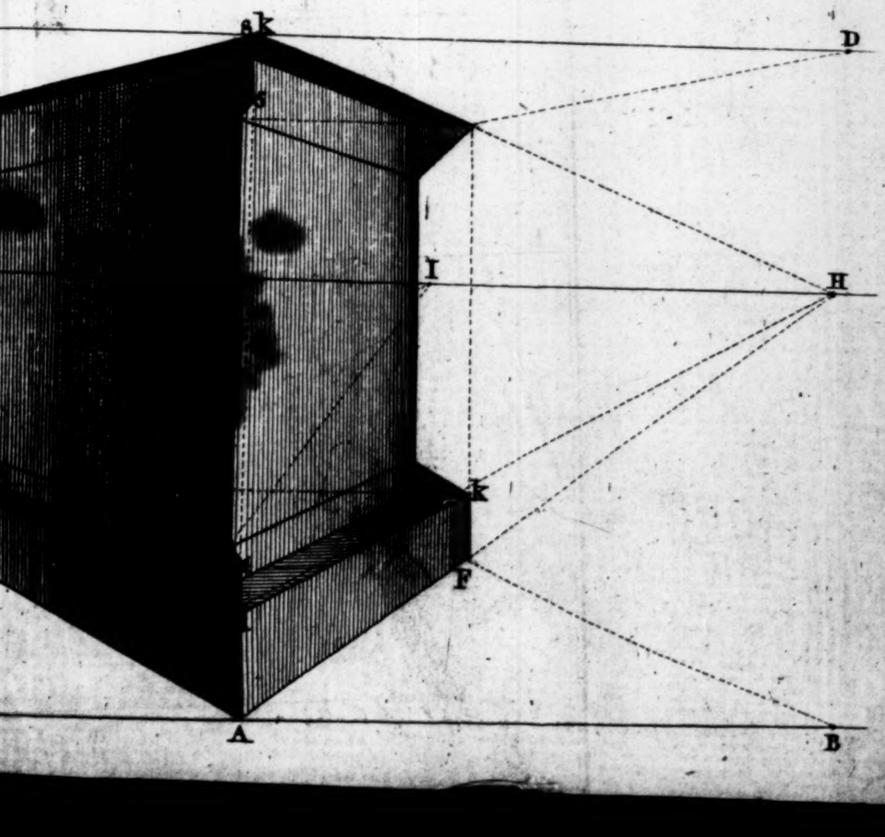

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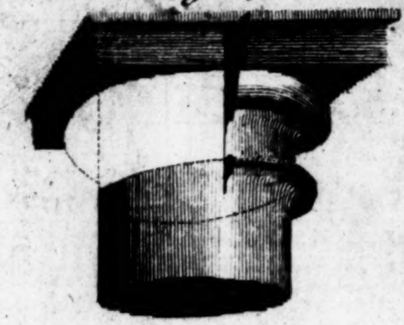

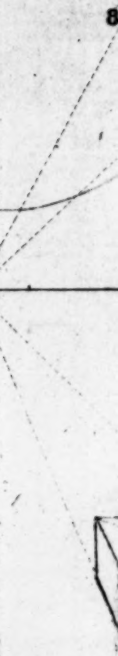

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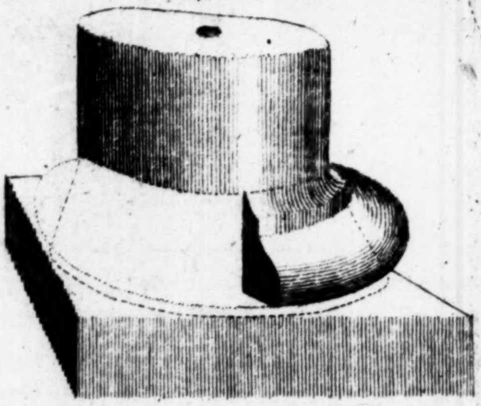

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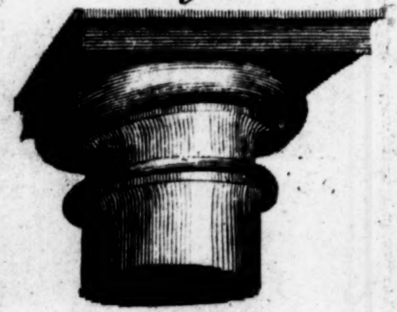

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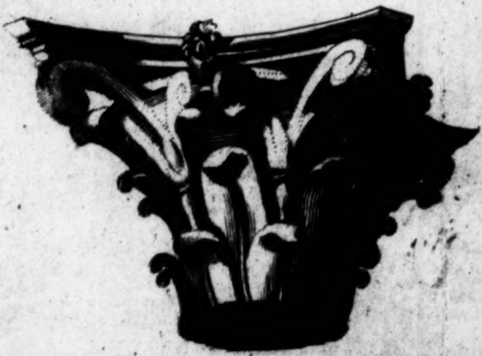

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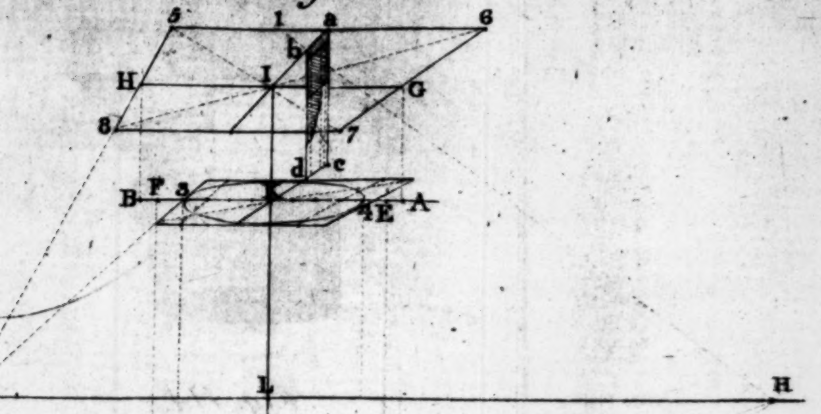

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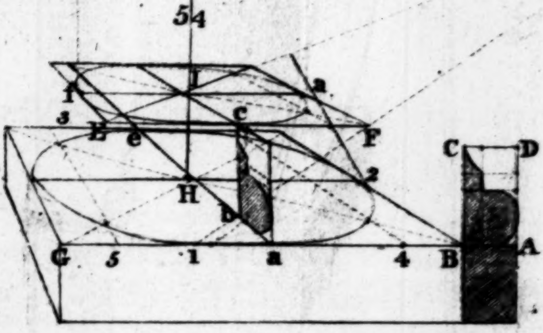

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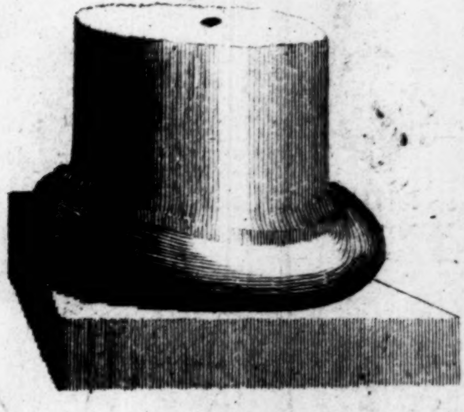

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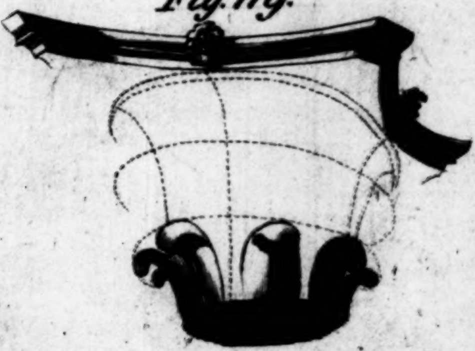

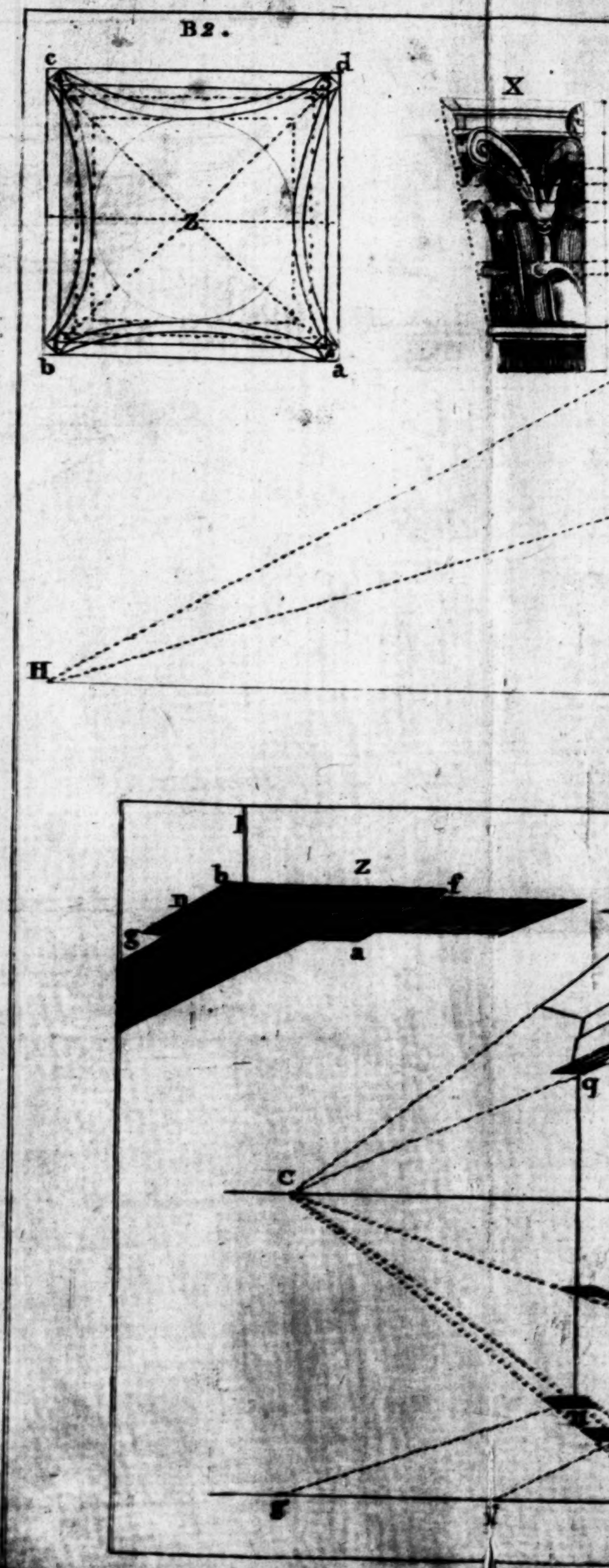
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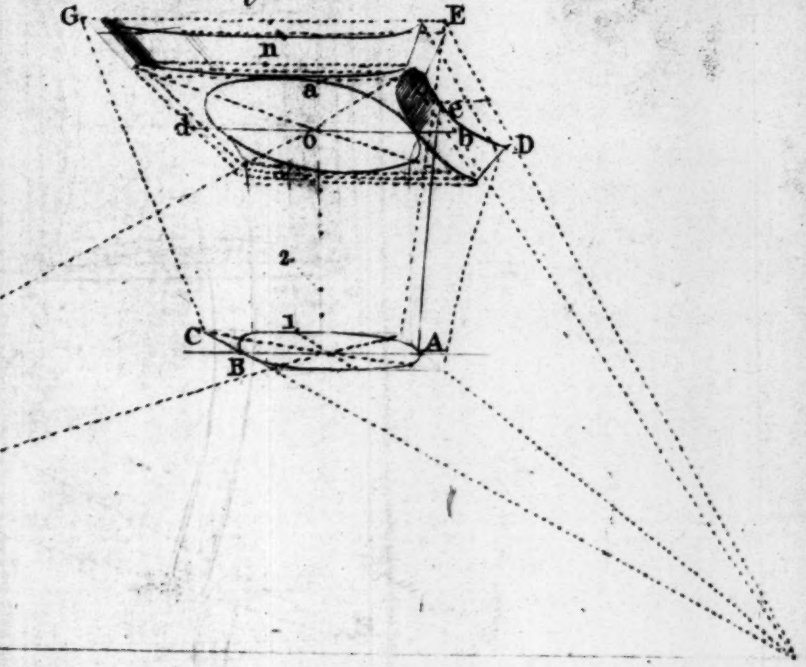
Fig. 121.



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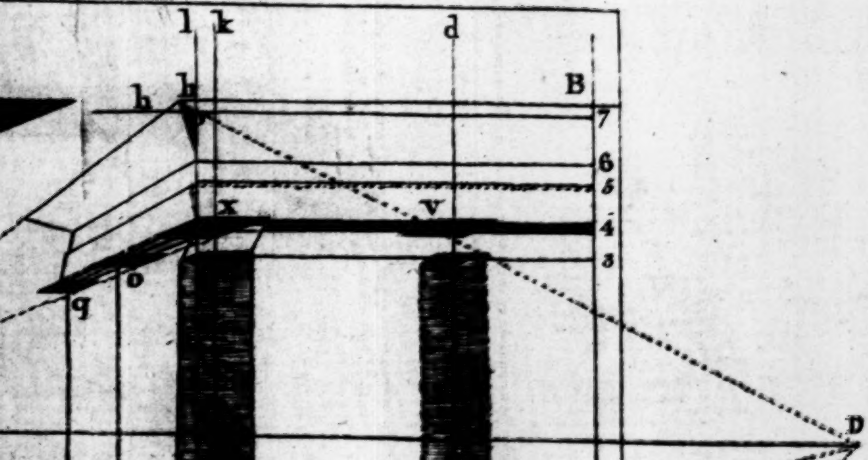

Fig. 123. *Plate 18.*

PENTAGRAPH.